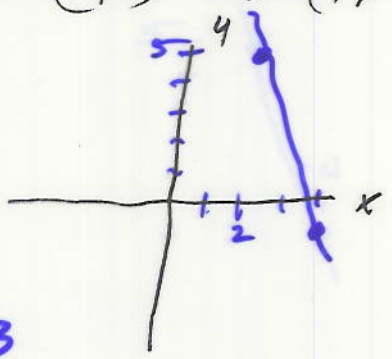
Math 201 Lecture #1

While I take roll, would you please do the following

1. For the line passing through the points (2,5) and (4,-1)

a) Graph the line in $\mathbb{R}^2$

x	y
2	5
4	-1



b) Give the slope

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-5}{4-2} = -3 = \frac{5-(-1)}{2-4} = -3$$

c) Give the eqn in point slope form

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -3(x-2)$$

d) Give the equation in general form: $ax + by = c$

$$y - 5 = -3x + 6$$

$$3x + y = 11$$

e) Give the equation in $y = mx + b$ form

$$y = -3x + 11$$

2. For the fun $f(x) = x^2 + 3x + 2$

a) Give the eqn of tan line @ $x = 2$

b) Find the area under f between $x = 1$ and $x = 4$.

$$f'(x) = 2x + 3$$

$$f'(2) = 7 = m$$

$$f(2) = 2^2 + 3 \cdot 2 + 2 = 12$$

$$y - 12 = 7(x-2)$$

2 b) $\text{Area} = \int_1^4 (x^2+3x+2)\,dx$

$= \frac{x^3}{3} + 3\frac{x^2}{2} + 2x \Big]_1^4$

$= \frac{64}{3} - \frac{1}{3} + \frac{3}{2}(16-1) + 2(4-1)$

$= 21 + \frac{45}{2}$

$= 21 + 22\frac{1}{2} + 6 = 49\frac{1}{2}$

M201 Lect #1 1/18/12

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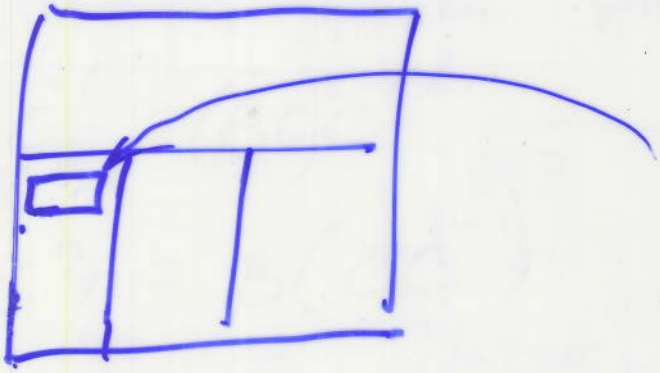
Calc 3 On Campus



Study Guide for Chap 13 for HW #1

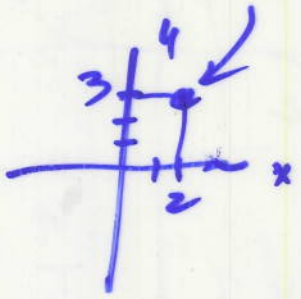
Today We'll do 13.1 & 13.2

§13.1

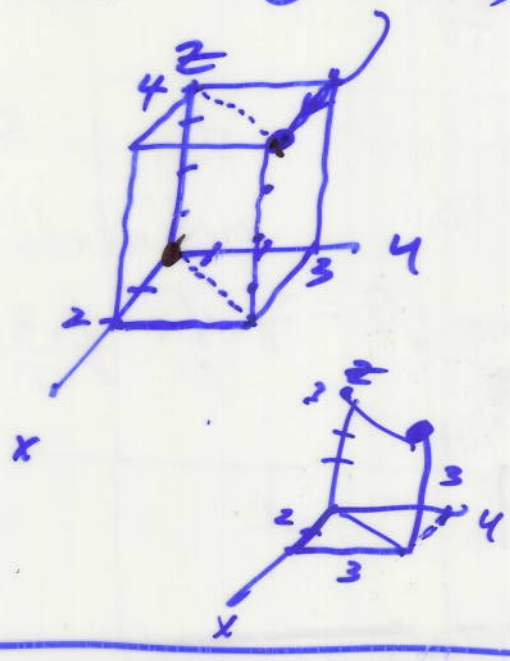
$\mathbb{R}^2$ $\mathbb{R}^3$

We generalize elementary ideas from $\mathbb{R}^2$ to $\mathbb{R}^3$

Point (2, 3)

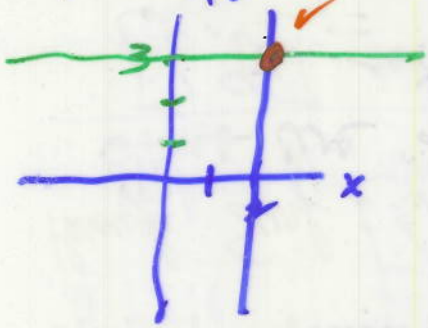


Point (2, 3, 4)



Simple Eqns.

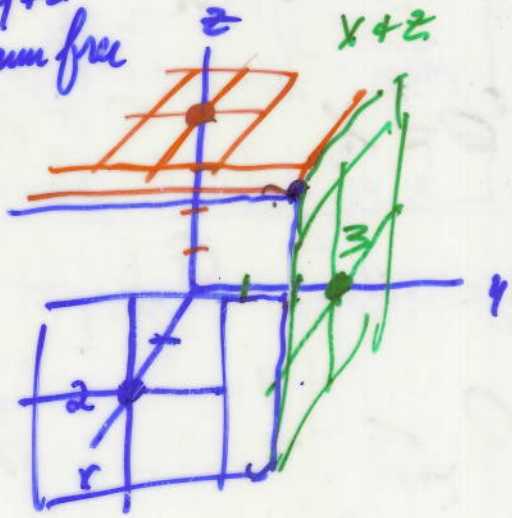
$x = 2$
4 runs, 4 free



$y = 3$
x runs free

$x = 2$
$y + z$ runs free

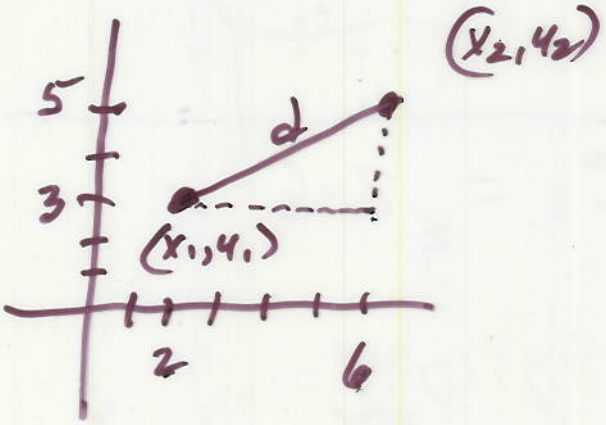
$y = 3$
$x + z$



$z = 4$
$x + y$ runs free

Distance

$\mathbb{R}^2$



$$d^2 = 4^2 + 2^2$$

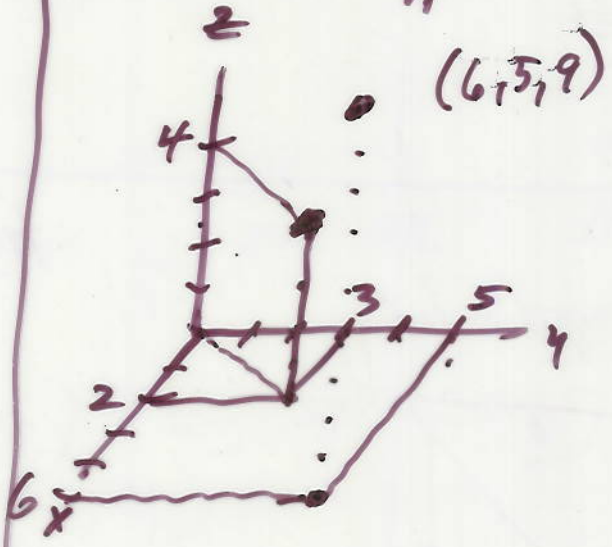
$$d^2 = (6-2)^2 + (5-3)^2$$

or more generally

$$d^2 = (x_2-x_1)^2 + (y_2-y_1)^2$$

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$\mathbb{R}^3$



guess

$$d^2 = (6-2)^2 + (5-3)^2 + (9-4)^2$$

$$h^2 = (6-2)^2 + (5-3)^2$$

$$d^2 = h^2 + (9-4)^2$$

$$d^2 = (6-2)^2 + (5-3)^2 + (9-4)^2$$

or in general

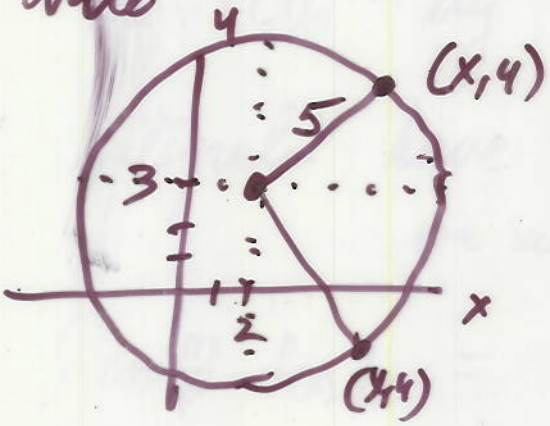
$$d^2 = (x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2$$

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

Let's use the dist^2 formula to create some harder eqns.

$\mathbb{R}^2$

Circle

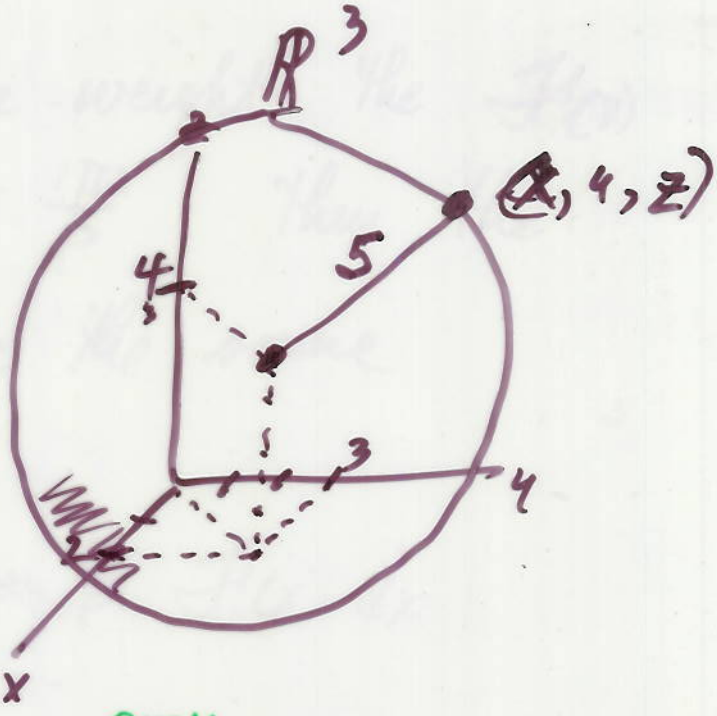


Use dist form. to form eqn

$(x-2)^2 + (y-3)^2 = 5^2$

$(x-h)^2 + (y-k)^2 = r^2$

$\mathbb{R}^3$



guess

$(x-2)^2 + (y-3)^2 + (z-4)^2 = 5^2$

or generally

$(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2 = r^2$

Standard form of a sphere.

Let's generalize again

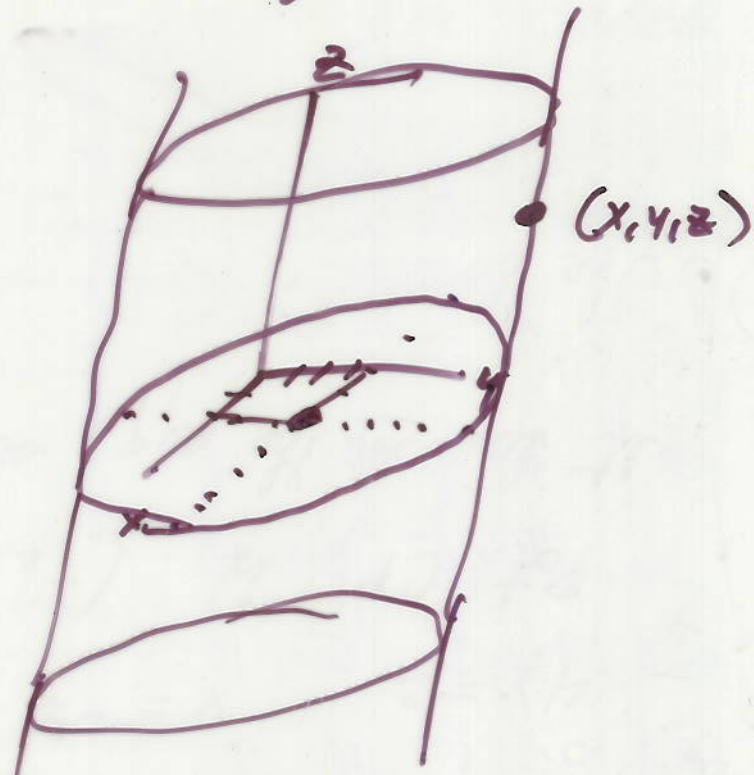
Circle $\mathbb{R}^2$ | _____ in $\mathbb{R}^3$

$(x-x_1)^2 + (y-y_1)^2 = r^2$

$(x-2)^2 + (y-3)^2 = 5^2$

$(x-2)^2 + (y-3)^2 = 5^2$

z runs free



right circular cylinder

$(z-4)(z-4) = z^2 - 4z - 4z + 16$ p6

Expand

$(x-2)^2 + (y-3)^2 + (z-4)^2 = 5^2$

$x^2 - 4x + 4 + y^2 - 6y + 9 + z^2 - 8z + 16 = 25$

$x^2 + y^2 + z^2 - 4x - 6y - 8z = -4$

Put into Standard form

$x^2 + y^2 + z^2 + 6x - 8y + 12z = 195$

$x^2 + 6x + 9 + y^2 - 8y + 16 \quad z^2 + 12z + 36 = 195 + 9 + 16 + 36$

$(x+3)^2 + (y-4)^2 + (z+6)^2 = 256 = 16^2$

Center is $(-3, 4, -6)$ Radius is 16

$\frac{6}{2}$	$\frac{8}{2}$	$\frac{12}{2}$
3	4	6
$3^2 = 9$	$4^2 = 16$	36

§13.2 Vectors

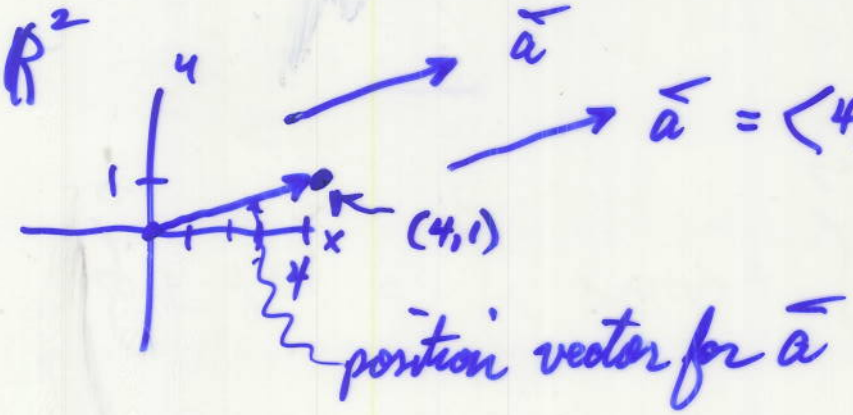
A Vector is a quantity having magnitude + direction

The Rule of Three

To learn math well do each topic

algebraically, numerically & graphically

$\mathbb{R}^2$



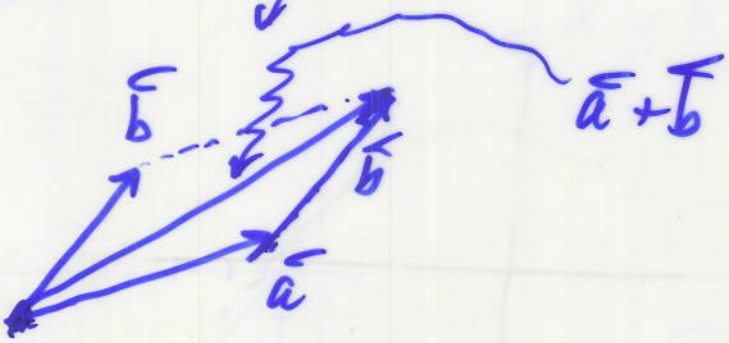
$\vec{a}$

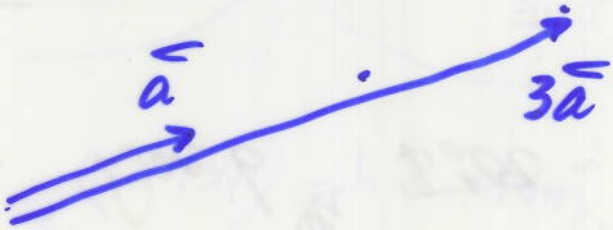
$\vec{a} = \langle 4, 1 \rangle$

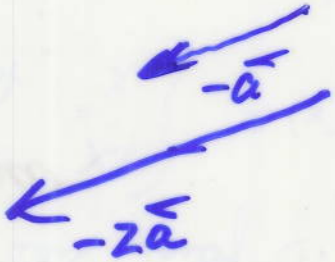
$\vec{a} = \langle a_1, a_2 \rangle$

For $\mathbb{R}^3$ $\quad \vec{a} = \langle a_1, a_2, a_3 \rangle$

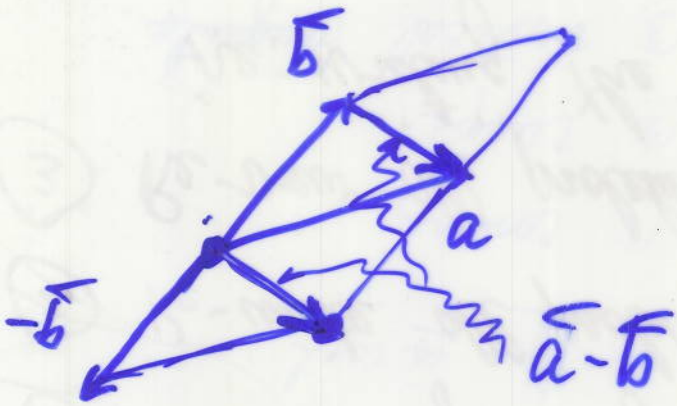
Operations: Addition + Scalar Mult + Linear Combinations p 8



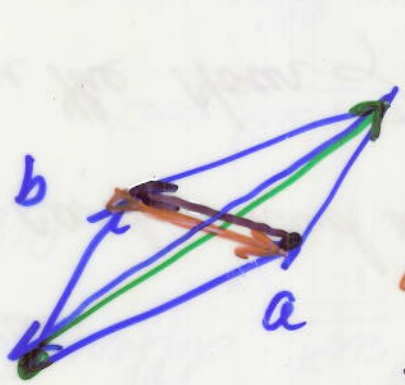




Subtraction $\vec{a}-\vec{b} = \vec{a}+(-\vec{b})$



So



Algebraically / Numerically

$\bar{a} = \langle 2, 6, 1 \rangle$

$\bar{b} = \langle 4, 0, -3 \rangle$

$c = 7$

$d = -2$

Linear combination

$$c\bar{a} + d\bar{b} = 7\bar{a} - 2\bar{b}$$

$$= 7\langle 2, 6, 1 \rangle - 2\langle 4, 0, -3 \rangle$$

$$= \langle 14, 42, 7 \rangle - \langle 8, 0, -6 \rangle$$

$$= \langle 6, 42, 13 \rangle$$

Algebraically

$$= c\langle a_1, a_2, a_3 \rangle + d\langle b_1, b_2, b_3 \rangle$$

$$= \langle ca_1, ca_2, ca_3 \rangle + \langle db_1, db_2, db_3 \rangle$$

$$= \langle ca_1 + db_1, ca_2 + db_2, ca_3 + db_3 \rangle$$

Magnitude of $\vec{a}$

$\vec{a} = \langle 2, 3, 4 \rangle$

$|\vec{a}|$ = magnitude = length

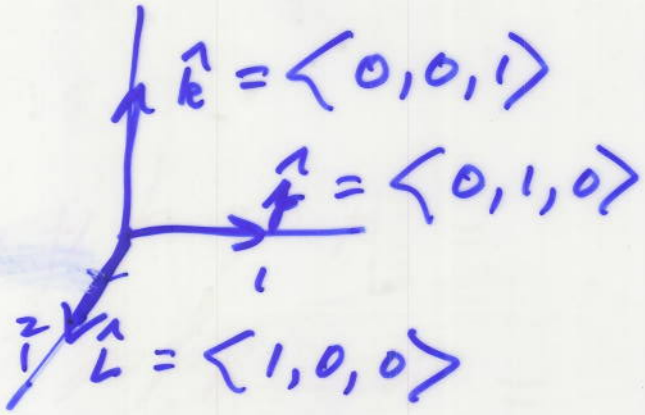
$= \sqrt{2^2+3^2+4^2} = \sqrt{29}$

A unit vector in the dir of $\vec{a}$ is

$$\frac{1}{|\vec{a}|} \cdot \vec{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\langle 2,3,4 \rangle}{\sqrt{2^2+3^2+4^2}} = \frac{\langle 2,3,4 \rangle}{\sqrt{29}}$$

$$= \left\langle \frac{2}{\sqrt{29}}, \frac{3}{\sqrt{29}}, \frac{4}{\sqrt{29}} \right\rangle$$

The three most popular unit vectors



$$\vec{a} = \langle 2,3,4 \rangle =$$

$$= 2\langle 1,0,0 \rangle + 3\langle 0,1,0 \rangle + 4\langle 0,0,1 \rangle$$

$$= 2\hat{i} + 3\hat{j} + 4\hat{k}$$