M 201 Lect #6 E 2-6-12

Here is another useful way to create the eqns of VVF's (Space curves):

Finding the intersection of surfaces

Ex. $x + y + z = 10$

$x^2 + x + 3y - z = 7$

Find $\begin{cases} x = f(t) = x(t) \\ y = g(t) = y(t) \\ z = h(t) = z(t) \end{cases}$

① Eliminate some var (say z)

$$\begin{array}{r} x + y + z = 10 \\ x^2 + x + 3y - z = 7 \\ \hline x^2 + 2x + 4y = 17 \end{array}$$

② Solve for elim. var (this time z)

$z = 10 - x - y$

③ "Paramize" ①

$x = t$

$4y = 17 - t^2 - 2t$

$y = \dfrac{17 - t^2 - 2t}{4}$

Eq of intersecting curve is

$$\begin{cases} x = t \\ y = \dfrac{17 - t^2 - 2t}{4} \\ z = 10 - t - \dfrac{17 - t^2 - 2t}{4} \end{cases}$$

Ex 2

$x+y+z=10$

$x^2+y^2=9$

Find curve of intersection

① ✓ $x^2+y^2=9$

② Solve for z $\quad z=10-x-y$

③ Parametrize #1 $\begin{cases} x = 3\cos t \\ y = 3\sin t \end{cases}$

④ $\begin{cases} x = 3\cos t \\ y = 3\sin t \\ z = 10-3\cos t-3\sin t \end{cases}$

$\vec{r}(t) = \langle 3\cos t, 3\sin t, 10-3\cos t-3\sin t \rangle$

The Calculus of VVFs (Space Curves)

For $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$

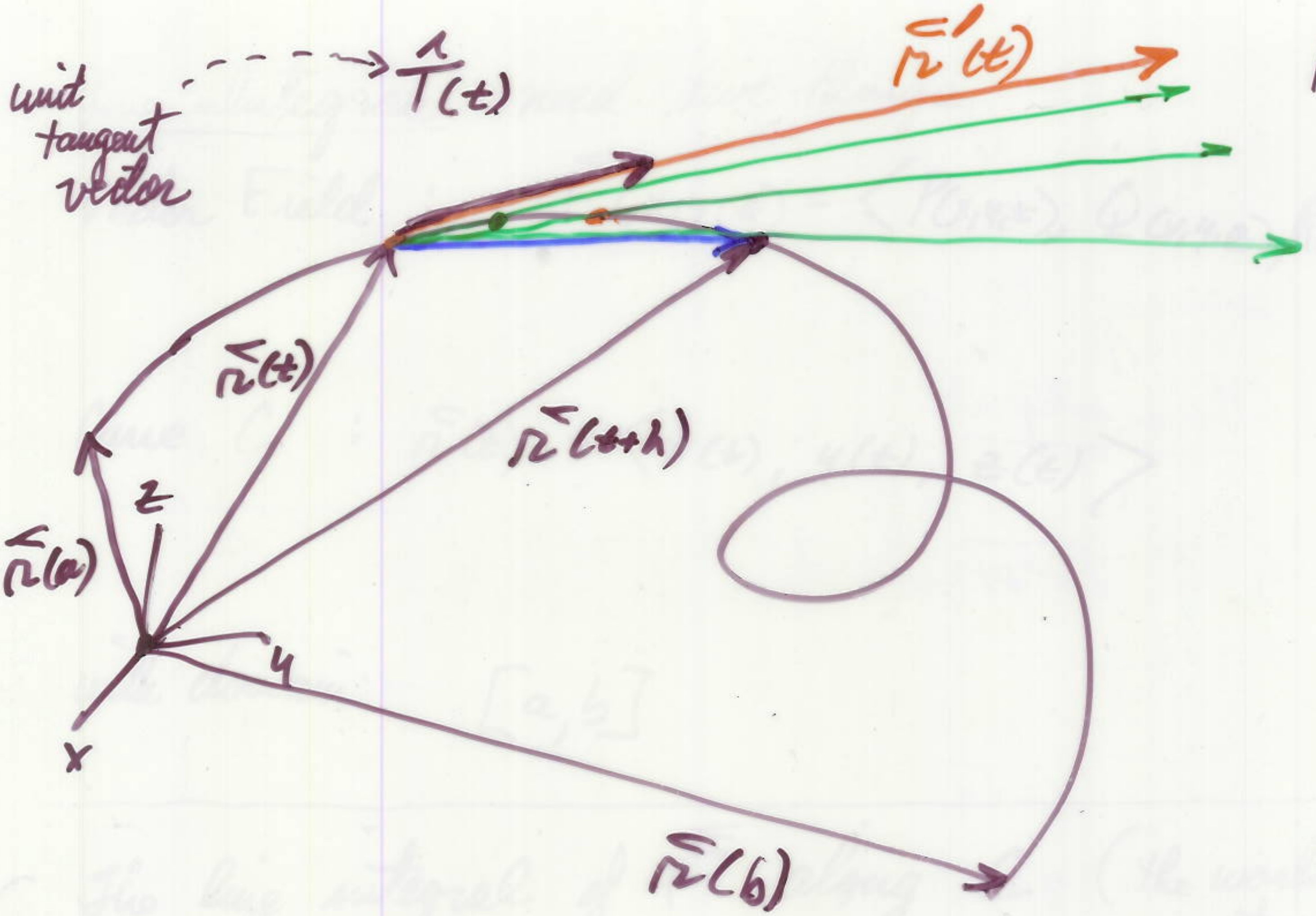
$$\lim_{t \to a} \vec{r}(t) = \left\langle \lim_{t \to a} f(t), \lim_{t \to a} g(t), \lim_{t \to a} h(t) \right\rangle$$

$$\frac{d}{dt}\vec{r}(t) = \vec{r}\,'(t) = \langle f'(t), g'(t), h'(t) \rangle$$

$$\vec{r}\,'(t) = \lim_{h \to 0} \frac{\vec{r}(t+h) - \vec{r}(t)}{h}$$

$$\int \vec{r}(t)\,dt = \left\langle \int f(t)\,dt, \int g(t)\,dt, \int h(t)\,dt \right\rangle$$

$$\int_a^b \vec{r}(t)\,dt = \left\langle \int_a^b \text{''}, \int_a^b \text{''}, \int_a^b \text{''} \right\rangle$$



$$\hat{T}(t) = \frac{\bar{r}'(t)}{|\bar{r}'(t)|}$$