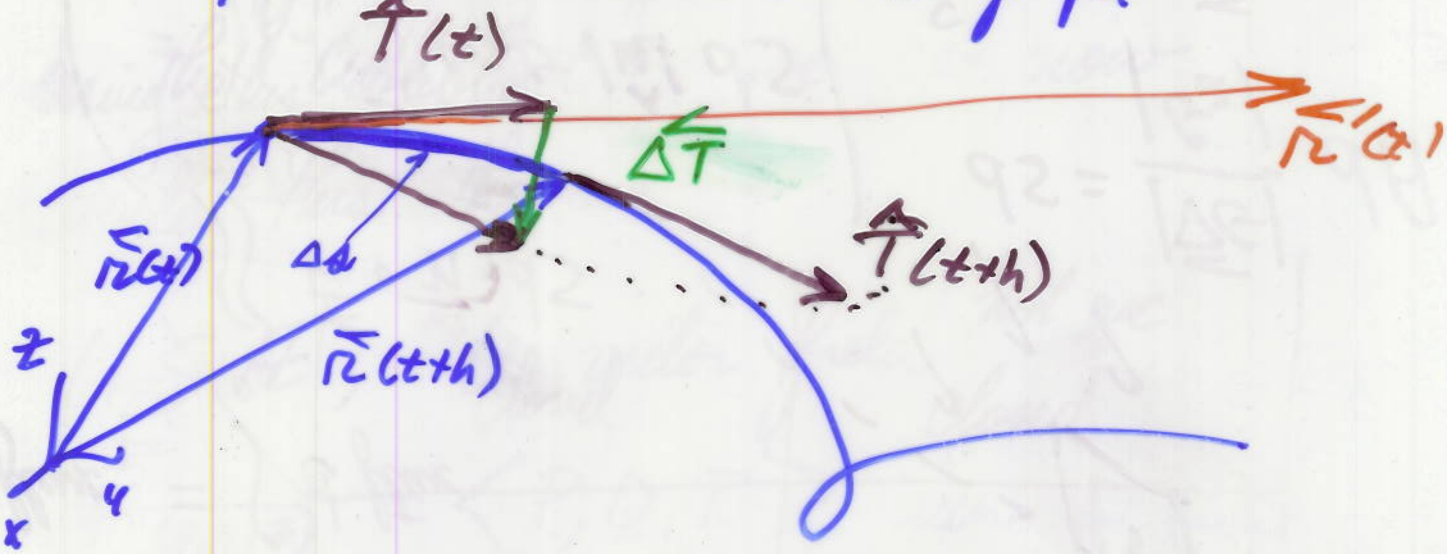
M 201 Lect #8 E 2-13-12

Let's depict the curvature on a graph



$$\left|\frac{\Delta\vec{T}}{\Delta s}\right| = \left|\frac{\hat{T}(t+h)-\hat{T}(t)}{\Delta s}\right|$$

$$K = \left|\frac{d\vec{T}}{ds}\right| = \left|\lim_{\Delta s\to 0}\frac{\Delta\vec{T}}{\Delta s}\right|$$

Aside

$$y = f(x) \qquad x = g(t)$$

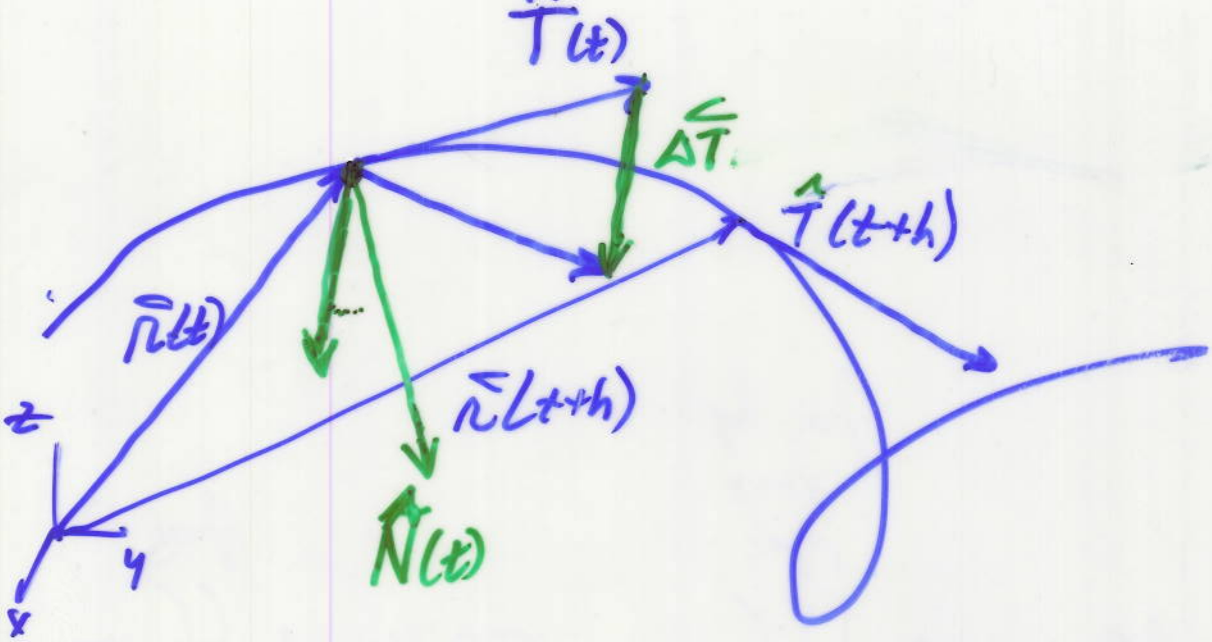
$$y = f(g(t))$$

want $$\frac{dy}{dt} = \frac{dy}{dx}\,\frac{dx}{dt} = \frac{dy}{dx_1}\,\frac{dx_1}{dx}\,\frac{dx_2}{dx_3}\,\frac{dx_3}{dt}$$

$$= f'(x)\cdot g'(t)$$

$$= f'(g(t))\cdot g'(t)$$

$$\frac{dy}{dt} = \frac{\frac{dy}{dx}}{\frac{dt}{dx}} = \frac{dy}{dx}\cdot\frac{dx}{dt}$$



$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

$$\lim_{\Delta t \to 0} \frac{\vec{\Delta T}}{\Delta t} = \frac{d\vec{T}}{dt}$$

$$= \vec{T}'(t)$$

$$\hat{N}(t) = \frac{\vec{T}'(t)}{|\vec{T}'(t)|}$$

$$K = \frac{|\vec{T}'|}{\frac{ds}{dt}} \qquad |\vec{T}'(t)| = \frac{ds}{dt} K$$

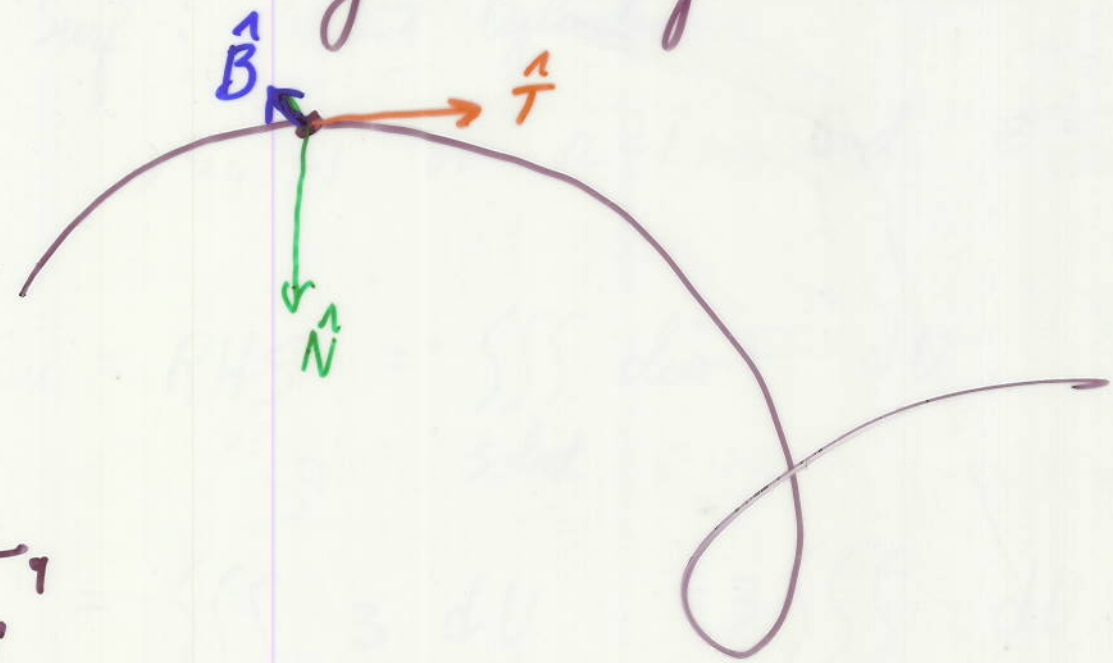
$$\boxed{\hat{N}(t) = \frac{\vec{T}'(t)}{\frac{ds}{dt} K}}$$

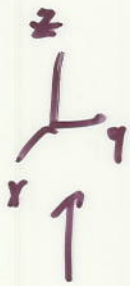
unit normal vector

$$\vec{T}'(t) = \frac{ds}{dt} K \hat{N}(t)$$

Define $\hat{B}(t) = \hat{T}(t) \times \hat{N}(t)$

These three vectors $\hat{T}(t), \hat{N}(t), \hat{B}(t)$
form a moving coord system.





base coord sys

__Ex__ Let's calculate some of the quantities for the twisted cubic

$$\vec{r}(t) = \langle t, t^2, t^3 \rangle$$

$$\vec{r}'(t) = \langle 1, 2t, 3t^2 \rangle$$

$$|\vec{r}'(t)| = \sqrt{1+4t^2+9t^4} = \frac{ds}{dt}$$

$$\hat{T}(t) = \frac{\langle 1, 2t, 3t^2 \rangle}{\sqrt{1+4t^2+9t^4}} = \frac{\vec{r}'(t)}{|\vec{r}'(t)|}$$

$$= \left\langle \frac{1}{\sqrt{1+4t^2+9t^4}}, \frac{2t}{\sqrt{1+4t^2+9t^4}}, \frac{3t^2}{\sqrt{1+4t^2+9t^4}} \right\rangle$$

$$\vec{T}'(t) = \frac{\sqrt{}\,\langle 0, 2, 6t \rangle - \langle 1, 2t, 3t^2 \rangle \frac{1}{2} \frac{1}{\sqrt{}} \cdot (0+8t+36t^3)}{\sqrt{}^{\,2}}$$

Now for the rest of the chapter we let t be time and $\vec{r}(t)$ have length units

$$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle \qquad t \in [a,b]$$

We wish to find velocity and acceleration in the $\hat{i}, \hat{j}, \hat{k}$ coordsystem and the $\hat{T}\,\hat{N}\,\hat{B}$ coord system.

$$\vec{v}(t) = \vec{r}\,'(t) = \langle x'(t), y'(t), z'(t) \rangle$$

$$\vec{a}(t) = \vec{v}\,'(t) = \vec{r}\,''(t) = \langle x''(t), y''(t), z''(t) \rangle$$

Recall $\hat{T} = \dfrac{\vec{r}\,'}{|\vec{r}\,'|} = \dfrac{\vec{r}\,'}{\frac{ds}{dt}}$

$$\boxed{\vec{v}(t) = \vec{r}\,'(t) = \frac{ds}{dt}\hat{T}(t) + 0\,\hat{N}(t) + 0\,\hat{B}(t)}$$

$$\vec{a}(t) = \frac{d}{dt}\vec{v}(t) = \frac{d}{dt}\left(\frac{ds}{dt}\hat{T}(t)\right)$$

$$= \frac{ds}{dt}\vec{T}'(t) + \frac{d^2s}{dt^2}\cdot\hat{T}(t)$$

$$= \frac{d^2s}{dt^2}\hat{T}(t) + \frac{ds}{dt}\vec{T}'(t)$$

$$= \frac{d^2s}{dt^2}\hat{T}(t) + \frac{ds}{dt}\frac{ds}{dt}K\hat{N}(t) + 0\hat{B}(t)$$

$$= \underbrace{\frac{d^2s}{dt^2}}_{a_T}\hat{T}(t) + \underbrace{\left(\frac{ds}{dt}\right)^2 K}_{a_N}\hat{N}(t) + 0\hat{B}(t)$$

a_T ↑ tangential comp of accel

a_N ↑ normal comp of accel

$$\vec{a}(t) = a_T\hat{T}(t) + a_N\hat{N}(t) + 0\hat{B}(t)$$