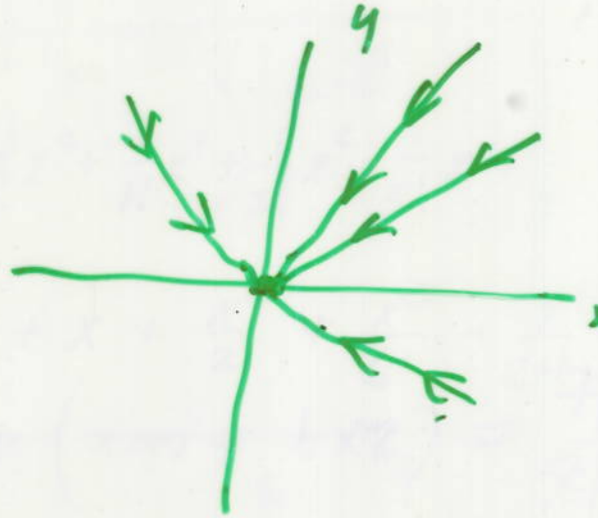
M 201 Lect # 10 E 2-20-12

$$\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4+y^2}$$



along $y = mx$

$y = mx + \cancel{b}^{\,0}$

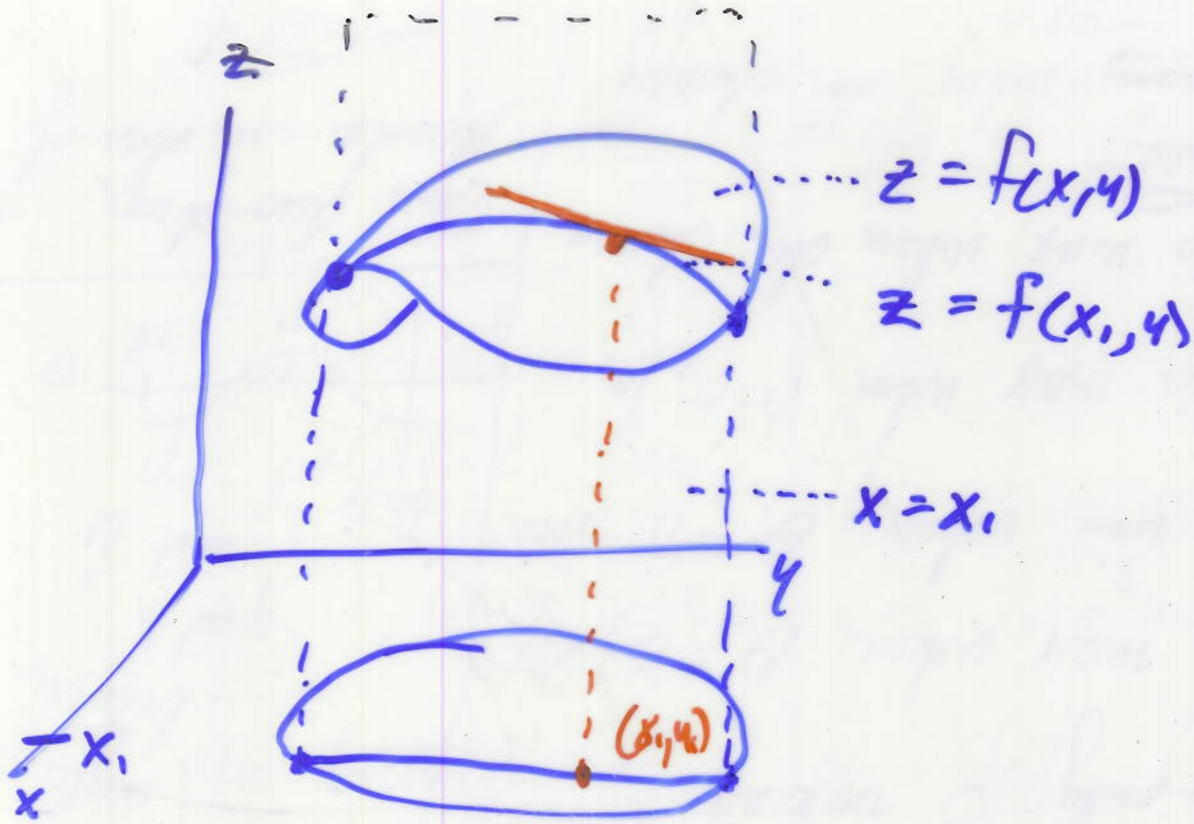
$y = mx$

$m \neq 0$

$$= \lim_{x\to 0} \frac{x^2 \cdot mx}{x^4 + (mx)^2}$$

$$= \lim_{x\to 0} \frac{\cancel{x^3}^{\,1} m}{\cancel{x^2}(x^2+m^2)}$$

$$= \lim_{x\to 0} \frac{mx \to 0}{x^2+m^2 \to m^2 \neq 0} = 0$$



a curve in a plane so that we can do Calc I to it.

fixed

$$\frac{d}{dy} f(x_1, y) = \frac{\partial}{\partial y} f(x_1, y) = f_y(x_1, y)$$

Again remove the 1.

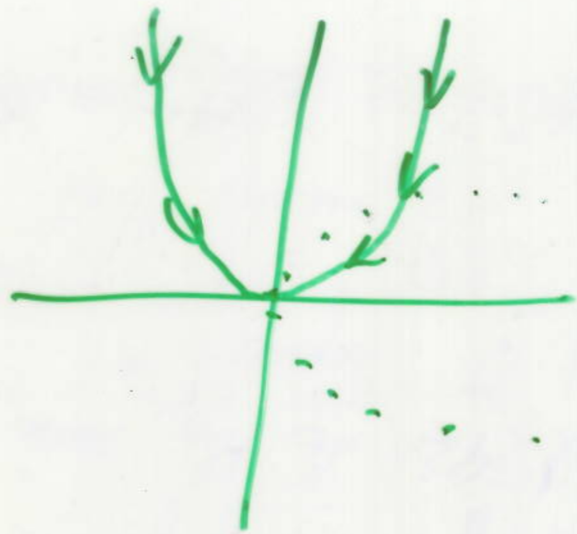
$$= \lim_{h \to 0} \frac{f(x_1, y+h) - f(x_1, y)}{h}$$

This $f_y(x,y)$ is called the

Partial Derivative of f w/r y.

$$\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$$

along $y=x^2$



$$= \lim_{x\to 0} \frac{x^2x^2}{x^4+x^4}$$

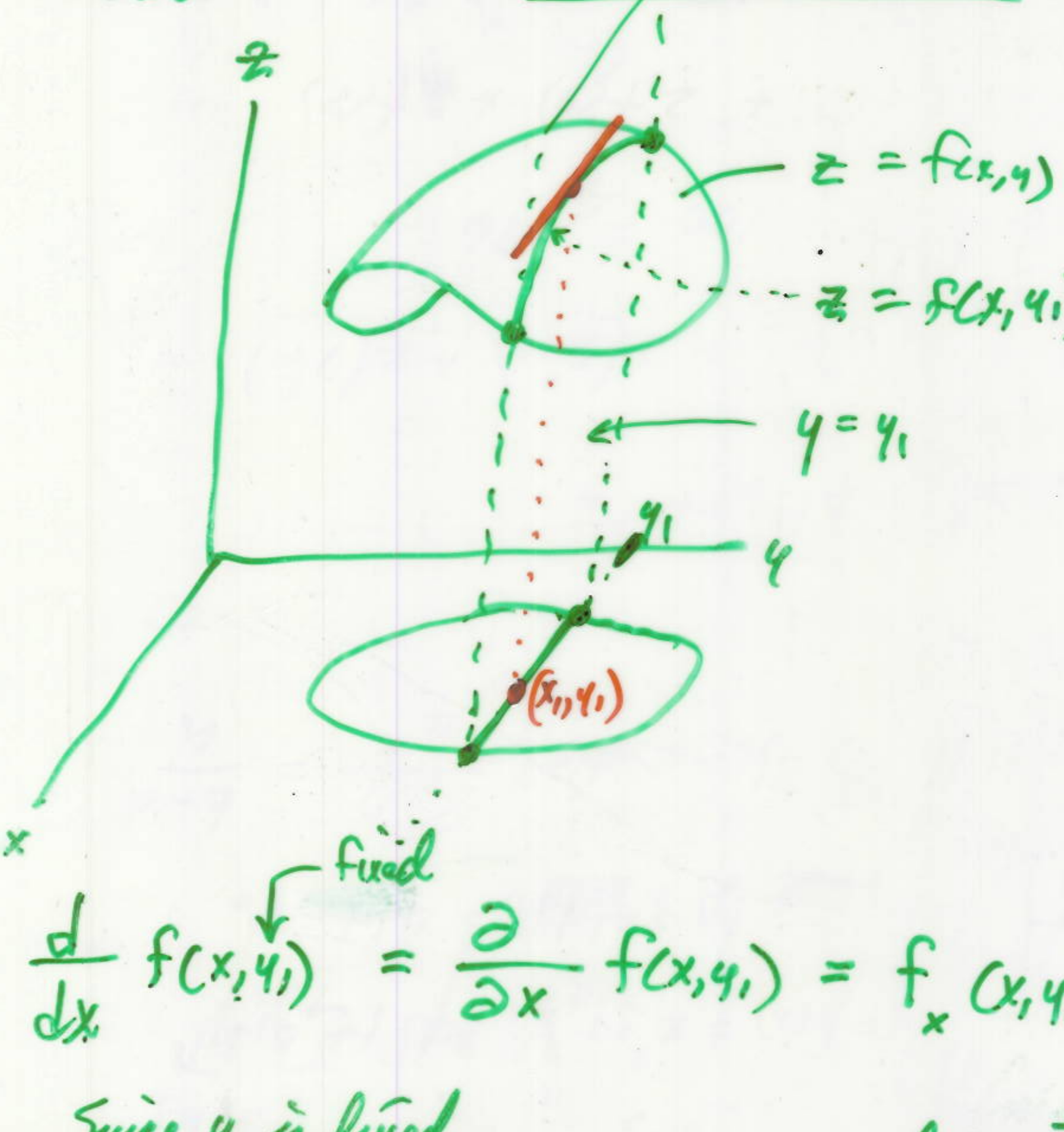
$$= \lim_{x\to 0} \frac{x^4}{x^4(1+1)} = \lim_{x\to 0} \frac{1}{2} = \frac{1}{2}$$

Since $0 \neq \frac{1}{2}$ We conclude limit DNE (does not exist)

One goal today is for a surface $z = f(x,y)$ find its tanplane's eqn.

Picture

Partial Derivatives



a curve lying in a plane so we can do Calc I with it

$$\frac{d}{dx} f(x, \overset{\text{fixed}}{y_1}) = \frac{\partial}{\partial x} f(x, y_1) = f_x(x, y_1)$$

Since y_1 is fixed we just remove the 1.

$$= \lim_{h \to 0} \frac{f(x+h, y_1) - f(x, y_1)}{h}$$

$f_x(x,y)$ is the partial derivative of f w/r x.

$$z = f(x,y) = x^7 + x^2y^5 + \cos y + 5$$

$$f_x(x,y) = 7x^6 + 2xy^5 + 0 + 0$$

$$f_y(x,y) = 0 + x^2 5y^4 - \sin y + 0$$

To get eqn of tan plane at (x_1, y_1, z_1)
we match its partials with the surface's partials

Eq of a plane $\quad a(x-x_1) + b(y-y_1) + c(z-z_1) = 0$

... becomes

$$z = z_1 + A(x-x_1) + B(y-y_1)$$

$$\frac{\partial z}{\partial x} = A \overset{set}{=} f_x(x_1, y_1)$$

$$\frac{\partial z}{\partial y} = B = f_y(x_1, y_1)$$

So eq of tan line is

$$z = f(x_1,y_1) + f_x(x_1,y_1)(x-x_1) + f_y(x_1,y_1)(y-y_1)$$