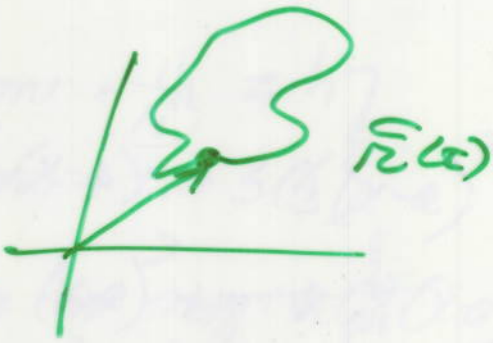
M 201 Lect #23 E 4-11-12

Green's "QP" Th (for closed curves

① V.F.: $\vec{F} = \langle P, Q, R \rangle$ ← on running

② Curve (Closed) . . . ⟶



$\vec{r}(a) = \vec{r}(b)$

" " "

Work = Circulation = Circ

$$= \oint_C \vec{F} \cdot d\vec{r} = \oint_C \langle P, Q \rangle \cdot \langle dx, dy \rangle$$

Green's QP Th

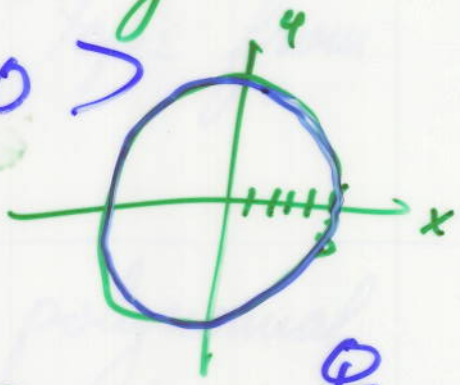
$$= \oint_C P\,dx + Q\,dy = \iint_{\text{reg interior of } C} (Q_x - P_y)\,dA$$

Ex Use Green's "QP" Th to find the circ of $\vec{F}$ about C where

$$\vec{F} = \langle P, Q \rangle = \langle 3y + e^{\sin x}, 7x + \sqrt{y^4+1} \rangle$$

and C is circle of rad 5 about origin

$$C: \vec{r}(t) = \langle 5\cos t, 5\sin t, 0 \rangle$$



$$\text{Circ} = \oint_C \vec{F} \cdot d\vec{r} = \int \langle P, Q \rangle \cdot \langle dx, dy \rangle$$

$$= \oint_C P\,dx + Q\,dy = \oint_C \left(3y + e^{\sin x}\right)dx + \left(7x + \sqrt{y^4+1}\right)dy$$

← Gr QP Th

$$= \iint_{\text{disk}} \left(Q_x - P_y\right) dA = \iint_{\text{disk}} (7 - 3)\, dA$$

$$= 4 \iint_{\text{disk}} dA = 4 \cdot \pi 5^2 = 100\pi$$

$$= 4 \int_{\theta=0}^{2\pi} \int_{r=0}^{5} r\, dr\, d\theta$$