Fundamental Dimensions and Units

Dimension (Sym)	Units: mks & cgs [Imperial]{comment}
Time (T)	second=s; minute; hour; year=31,557,600 sec {365.25·24·60·60}
Distance (D,λ)	meter; centimeter=cm; kilometer=km [m≈3.28feet; km≈0.621mile] {λ=wavelength uses Ångstrom=Å=10^{-10}m}
Mass (M)	kilogram=kg; gram=gm; tonne=1000kg [kg≈0.0685slug≈2.20lbM]
Charge (Q)	Coulomb= q or c {q≈$6.2415\cdot10^{18}e^{-}$}
Temperature	°Kelvin {Absolute Zero=0°K ≈ -273°C ≈ -460°F}
Luminosity	Candela (= Lumen per steradian)

There are other measures and units not associated to dimensions:

Mole	# of molecules given by Avogadro's # ≈ $6.02\cdot10^{23}$
Percent(%)	Dimensionless #·100
Radian	Dimensionless # = measure of an angle
Steradian	Dimensionless # = measure of a solid angle

Only quantities with the same units may be added or subtracted. When multiplying or dividing quantities with units, multiply or divide the units. Units are never ignored or destroyed. When taking a square root of a quantity with units, those units must be squared. Quantities which are input into transcendental functions (sin, tan, arcsin=$\sin^{-1}$, $\tan^{-1}$, exp=e^, a^, ln, $\log_b$, erf, etc.) must be dimensionless.

Slope $y' = {}^{dy}/_{dx}$; units(y)/units(x) = rate of change

Curvature ≈ y'' ; units(y)/units(x)^2
{ more properly it's $y'' \div (1 + y'^2)^{3/2}$), but only when units(y)=units(x) }

Derived Dimensions and Units

Dimension (Sym)	Units: mks & cgs	[Imperial]
Frequency (#/T)	Hertz=Hz= per second = /s = s^{-1}	
Angular-Velocity (ω)	Hertz=(radians)/s	
Velocity (v=L/T)	m/s; cm/s; km/hr	[ft/s; mi/hr]
Acceleration (a=v/T)	m/s^2; cm/s^2	
Force (F=Ma)	Newton=$kg\cdot m/s^2$; dyne=$gm\cdot cm/s^2$=10^{-5}N	[N≈0.225lbF]
Energy ($E=Mv^2$)	Joule=N·m=$kg\cdot m^2/s^2$; kJ=1000J; erg=dyne·cm;	
Work (Mv^2=FD)	KiloWatt-Hour=3600kJ	[J≈0.735ftlb]
Heat	calorie (J≈0.239cal)	[kJ≈0.947BTU]
Torque	J=N·m(per radian)	
Power (W=E/T)	Watt=Joules/sec= $kg\cdot m^2/s^3$; kW=1000W	[kW≈1.32HorsePower]
Current (I=Q/T)	Ampere=Coulombs/sec=q/s	
Voltage (V=J/Q)	Volt=Joules/Coulomb= $kg\cdot m^2/(s^2\cdot q)$	
Capacitance	Farad=Coulombs/Volt= $s^2\cdot q^2/(kg\cdot m^2)$	
Resistance	Ohm=Volts/Amp=$kg\cdot m^2/(s\cdot q^2)$	
Inductance	Henry=Volt·sec/Amp=J/Amp^2=$kg\cdot m^2/q^2$	

Prefix Factor	Divide	Times
10	deci	deca
100	centi	hecto
1000=10^3	milli	kilo
10^6	micro	mega
10^9	nano	giga
10^{12}	pico	tera
10^{15}	femto	peta
10^{18}	atto	exa

Definitions, Origins and Conversions

Second was originally $^1/_{86,400}$ (24·60·60) of mean solar day. It is now 9,192,631,770 vibrations of a Cs_{133} atom in ground state.

Meter was originally $^1/_{10,000,000}$ the distance from North Pole to Equator and later the length of a standard Pt-Ir bar. It is now the distance traveled in a vacuum by light during $^1/_{299,792,458}$ second.

Mile (5280 ft ≈ 1.61 km) was originally 1000 running strides (≈5.3ft).

Gram was originally the mass of 1cc (cm^3) of H_2O at ≈4°C. Now, one kilogram is the mass of a standard Pt-Ir cylinder. It is proposed that a gram be redefined so 12 grams equal precisely the mass of a mole of C_{12}.

Coulomb is precisely an Ampere·second. **Ampere** was originally the amount of current required to produce a force of two dynes/cm between two wires one cm apart. It may soon be redefined as a current of $6.2415093 \cdot 10^{18}$ electrons per second.

Temperature measures **quality of heat**, which determines rate and direction of **heat flow**. **Quantity of heat** is energy, usually measured in calories. A calorie (≈4.184J) was originally the heat needed to raise the temperature of 1cc of H_20 by 1°C. A Calorie=1000cal. A BTU (≈1056J) was originally the heat needed to raise 1lbM H_20 by 1°F.

Acceleration of gravity = g ≈ 9.807 m/s^2 ≈ 32.17 ft/s^2
lbF = slug ·1ft/s^2 ≈ 14.59kg·0.3048m/s^2 ≈ 4.447N
lbM = lbF÷g = (slug ·1ft/s^2)/(32.17ft/s^2) ≈ $^1/_{32.17}$ slug ≈ 0.4536 kg

FootPound = ft·lb = work done by a pound of force exerted over a distance of one foot ≈ 0.3048m· 4.447N ≈ 1.356J.

HorsePower = HP = 550 ft·lb/s ≈ 550·1.356J/s ≈ 756W = 0.756kW.
(I can do 550 ft-lb of work: lift 550 pounds 1 foot high using a pulley. Unlike a horse, I lack the power to do it in 1 second.)

Speed of light = c = 299,792,458 m/sec ≈ 300,000 km/s ≈ 186,000 mi/sec.
Light Year is the distance traveled by light through a vacuum in a year, which is ≈ $9.4607 \cdot 10^{12}$ km.

Scholium: What is a Dimension?

Why are moles just units, but coulombs are fundamental units for the fundamental dimension of charge? Both are counts: molecules or electrons. For that matter, a second is a count of vibrations, a meter could be a count of wavelengths, a gram a count of atoms and a lumen a count of photons. Why is each a measure for a fundamental dimension?

A mole is truly just a count of discrete objects. But charge is a property of not just electrons, but of many fundamental particles, including protons, positrons, and quarks. Charge is preserved even when the particles are not. Pions, muons and electrons may transition one to the other. The charge is conserved but the particles are not. (see: http://www.av8n.com/physics/one-kind-of-charge.htm) Similarly, mass measures a property common to many particles. The color-charge of the nuclear force could be called a seventh fundamental dimension.

Distance and time measure more abstract properties. Both are strongly correlated to human perception, as are temperature (hotness) and luminosity (brightness). Explicit definitions for distance and time lead to deep philosophical issues. Are they aspects of the physical world (Newton and Einstein) or the constructions of human thought (Kant)? In science, it is simpler to skirt such issues and define properties solely in terms of how they are measured. Such definitions are called operational. This is why most units are ultimately defined as counts of things, but the property thereby measured exists apart from the explicit things being counted.

In Thermodynamics and Statistical Mechanics, a theoretical definition of temperature, independent of human perception, is formulated. (see: http://www.princeton.edu/~achaney/tmve/wiki100k/docs/Thermodynamic_temperature.html) The temperature of a substance is related to the average kinetic energy of the atoms, but it is not a measure of energy.

Luminosity is explicitly a measure of brightness as perceived by humans (see: http://hyperphysics.phy-astr.gsu.edu/hbase/vision/bright.html). A candela is the brightness of a candle. A lumen is about $^{1}/_{683}$ Watt for monochromatic (λ=550 nanometers) green light, which is about $4.0 \cdot 10^{15}$ photons/sec. A mole/sec of such photons would be about $1.5 \cdot 10^{8}$ lumens $\approx$ 220 kW. For comparison, metal cutting lasers are about 1 kW.