Laws of Algebra

Operations: where **a, b, c, d** are **whole numbers**

Addition: **a + b (Repeated Counting)**

Commutative: **a + b = b + a** **a + 0 = a = 0 + a**

Associative: **a + (b + c) = (a + b) + c**

Multiplication (Repeated addition): **a × b = a·b = a*b = ab = a + a + ... +a (b times)**

Commutative: **ab = ba** **a × 0 = 0 = 0 × a**

Associative: **a(bc) = (ab)c**

Distributive: **a(b + c) = ab + ac = (b + c)a**

Exponentiation (Repeated multiplication): $a^b = a\text{^}b = a \times a \times ... \times a$ **(b times)**

Neither commutative nor associative

$a^b a^c = a^b \times a^c = a^{b+c}$ $a^0 = 1,\ 0^a = 0\ (a \neq 0)$

$(a^b)^c = a^{bc}$

$(ab)^c = a^c b^c$

Subtraction (Inverse addition): **a - b = c ⇔ a = b + c**

Neither commutative nor associative

Negatives: **a + -a = 0**

a - b = a + (-b) = -(b - a) **(subtract is add negative)**

-(-a) = a **(-1)a = -a**

-(ab) = (-a)b = a(-b) **(-a)(-b) = ab**

a(b - c) = ab - ac = (b - c)a **-(a + b) = -a + -b**

Division (Inverse multiplication): $a \div b = {}^a/_b = c \Leftrightarrow a = bc$

(Allow both negative and positive **a**'s and **b**'s, where $b \neq 0$**)**

Neither commutative nor associative

Fractions: $a \times ({}^1/_a) = 1$

$${}^a/_b = a \times ({}^1/_b)$$ **(divide is multiply by reciprocal)**

$$({}^a/_b)({}^c/_d) = {}^{ac}/_{bd}$$

$$({}^a/_b) \div ({}^c/_d) = ({}^a/_b)/({}^c/_d) = ({}^a/_b)({}^d/_c) = {}^{ad}/_{bc}$$

$$({}^a/_b) \pm ({}^c/_d) = {}^{(ad \pm bc)}/_{bd}$$ **(common denominators)**

Remainders: a ÷ b = c with remainder of **d** $(= c + {}^d/_b) \Leftrightarrow a = bc + d$

Precedence Rules:

First: Do Exponents, left to right;

Second: Do Multiplications and Divisions, left to right;

Third: Do Additions and Subtractions, left to right.

Parentheses override this normal order.

Solving Equations - Both Sides: Do the same operation on both sides!

Laws of Algebra

Operations: where **a, b, c, d** are **real numbers**

Addition: **a + b**

Commutative: **a + b = b + a** **a + 0 = a**

Associative: **a + (b + c) = (a + b) + c**

Multiplication **a × b = ab = a*b**

Commutative: **ab = ba** **a × 0 = 0**

Associative: **a(bc) = (ab)c**

Distributive: **a(b + c) = ab + ac = (b + c)a**

Exponentiation a^b = **a^b**

Where the **base a > 0** or the **exponent b** is an integer

Neither commutative nor associative

$a^b a^c = a^b \times a^c = a^{b+c}$ $a^0 = 1$, $0^a = 0$ $(a \neq 0)$

$(a^b)^c = a^{bc}$ $(ab)^c = a^c b^c$

Subtraction (Inverse addition): **a - b**

Neither commutative nor associative $a(b - c) = ab - ac$

Negatives: **a + -a = 0**

$-(-a) = a$ $-(a + b) = (-a + -b)$

$a + (-b) = a - b = -(b - a)$ $(-a)(-b) = ab$

$(-1)a = -a$ $(-a)b = a(-b) = -(ab)$

Division (Inverse multiplication): $a \div b = {}^a/_b$ (where $b \neq 0$)

Neither commutative nor associative ${}^{(a+b)}/_c = {}^a/_c + {}^b/_c$

Fractions: $a \times ({}^1/_a) = 1$ ${}^{(-a)}/_b = -({}^a/_b) = {}^a/_{(-b)}$

${}^a/_b = a \times ({}^1/_b)$ $({}^a/_b)/({}^c/_d) = {}^{ad}/_{bc}$

$({}^a/_b)({}^c/_d) = {}^{ac}/_{bd}$ $a^b/a^c = a^b \div a^c = a^{b-c}$

$({}^a/_b) + ({}^c/_d) = {}^{(ad+bc)}/_{bd}$ $({}^a/_b)^c = a^c/b^c$

Negative Exponents: $a^{-b} = {}^1/a^b = ({}^1/_a)^b$ $({}^1/_a)^{-b} = a^b$

Roots and Logarithms (Inverse exponentiation) (where **base > 0**)

Roots: $({}^b\sqrt{a})^b = {}^b\sqrt{(a^b)} = a$ ${}^c\sqrt{(ab)} = ({}^c\sqrt{a})({}^c\sqrt{b})$

Fractional Exponents: $(a^{1/b}) = {}^b\sqrt{a}$ $(a^{b/c}) = {}^c\sqrt{a^b} = ({}^c\sqrt{a})^b$

Logarithms: $b^{(\log_b a)} = a = \log_b(b^a)$ $\log_b(ac) = \log_b a + \log_b c$

$\log_b(a^c) = c \times \log_b a$ $\log_c(a) = \log_b a / \log_b c$

Precedence Rules:

First: Do Exponents, Roots and Logarithms (carefully);

Second: Do Multiplications and Divisions, left to right;

Third: Do Additions and Subtractions, left to right.

Parentheses override this normal order.