The Probabilities of Poker

The simplest way to calculate poker probabilities is to count in how many ways a given type of 5-card poker hand can be dealt. The probability of the given type of hand is then:

(# of ways to deal one) / (# of ways to deal 5 cards)

Cards are dealt one at a time, and how many ways a hand of given type can be dealt is just the product of how many ways each card can be dealt.

In how many ways can five cards be dealt? There are 52 cards.

52 ways to deal the first card;
51 ways remaining to deal the second card;
50 ways remaining to deal the third card;
49 ways remaining to deal the fourth card;
48 ways remaining to deal the fifth and last card.

There are 52×51×50×49×48 = 311,875,200 ways to deal five cards.

The order in which the cards are dealt does not affect the value of a poker hand (although it may very well affect the betting strategy). The two hands:

J♠, A♦, 5♥, 9♥, 2♣ and 5♥, 9♥, J♠, 2♣, A♦

are equally worthless and are really just the same hand sorted in a different order. We will call such hands different deals of the same combination.

In how many different orders may a given five card combination be dealt? The same strategy applies. There are:

5 ways to choose the first card;
4 ways to choose the second card;
3 ways to choose the third card;
2 ways to choose the fourth card;
1 way to choose the fifth card.

The are 5×4×3×2×1 = 120 ways to deal a specific combination.

There are hence:

311,875,200 / 120 = 2,598,960

different combinations of 5-card hands.

How many ways can a 5-card stud hand be dealt?

52	51	50	49	48

That's 52 × 51 × 50 × 49 × 48 = 311,875,200 (2,598,960 combinations)

How many ways can a straight flush be dealt in order?

40	1	1	1	1

First card must be A thru 10 – there's 40 of them.
That's 40 combos; 40 × 120 = 4800 ways to deal a straight flush.
Probability = 4800 / 311,875,200 = 1 / 64974 = 0.0000154

How many ways can four of a kind be dealt?

52	3	2	1	48
			48	1
		48	2	1
	48	3	2	1
		3	2	1

(Blank means same as above; same color means same rank)
Explanation by row:

1. **You are dealt a first card (52 ways). Your next 3 cards are the same rank. Your last card is something else (48 ways).**
2. **You are dealt a first card (52 ways). Your next 2 cards are the same rank. Your next card is something else (48 ways).**
3. **You are dealt a first card (52 ways). Your next card is the same rank. You next card is something else (48 ways).**
4. **You are dealt a first card (52 ways). Your next is something else (48 ways). Your next 3 cards match the first.**
5. **You are dealt a first card (52 ways). Your next is something else (48 ways). Your next 3 cards match the second.**

That's 52 × 48 × 5(3 × 2 × 1) = 74,880 (624 combinations)
Probability = 74,880 / 311,875,200 = 1 / 4165 = 0.0002401

How many ways can a full house be dealt?

52	3	2	48	3
		48	2	3
			3	2
				2
	48	3	2	3
			3	2
				2
		3	3	2
				2
			2	3

(Blank means same as above; same color means same rank)
That's 52 × 48 × 10(3 × 2 × 3) = 449,280 (3744 combinations)
Probability = 449,280 / 311,875,200 = 6 / 4165 = 0.00144

How many ways can a flush be dealt?

52	12	11	10	9

That's 52 × 12 × 11 × 10 × 9 = 617,760
Subtract the 4800 straight flushes: 612,960 (5108 combinations)
Probability = 612,960 / 311,875,200 = 1277 / 649740 = 0.001965

How many ways can a straight be dealt in order?

40	4	4	4	4

That's 40 × 4 × 4 × 4 × 4 = 10,240 combinations.
Subtract the 40 straight flush combinations: 10,200.
That's 10,200 × 120 = 1,224,000 in any order.
Probability = 1,224,000 / 311,875,200 = 5/1274 = 0.003925

How many ways can three of a kind be dealt?

52	3	2	48	44
		48	2	44
			44	2
	48	3	2	44
			44	2
		3	2	44
			44	2
		44	3	2
			3	2
			3	2

(Blank means same as above; same color means same rank)
That's 52 × 48 × 44 × 10(3 × 2) = 6,589,440 (54,912 combinations)
Probability = 6,589,440 / 311,875,200 = 88 / 4165 = 0.02113

How many ways can two-pair be dealt?

52	3	48	3	44
			44	3
				3
	48	3	3	44
			44	3
				3
		3	3	44
			44	3
				3
		44	3	3
				4
			3	3
				3
			3	3
				3

(Blank means same as above; same color means same rank)
That's 52 × 48 × 44 × 15(3 × 3) = 14,826,240 (123,552 combinations)
Probability = 14,826,240 / 311,875,200 = 198 / 4165 = 0.04754

How many ways can a pair be dealt?

52	3	48	44	40
	48	3	44	40
		3	44	40
		44	3	40
			3	40
			3	40
			40	3
				3
				3
				3

(Blank means same as above; same color means same rank)
That's 52×48×44×40×(10×3) = 131,788,800 (1,098,240 combinations)
Probability = 14,826,240 / 311,875,200 = 352 / 833 = 0.4226

How many ways can a bust be dealt?

52	48	44	40	36

That's 52×48×44×40×36 = 158,146,560.
Subtract straights, flushes and straight flushes:
- (1,224,000 + 612,960 + 4800) = 156,304,800 (1,302,540 combinations)
Probability = 156,204,800 / 311,875,200 = 0.5009