# Pythagorean Scale

**Pythagoras (circa 580-500BC) is usually credited with discovering that vibrating strings with lengths the ratios of small whole numbers of each other produced a pleasing harmony, especially the ratios 2:1, 3:2 and 4:3.**

**The Pythagorean scale ("Do, Re, Mi, Fa, So, La, Ti, Do" from The Sound of Music) is a tuning for the diatonic scale based on such harmonies and consists of only two intervals: 9:8 (a perfect second) and 256:243 (a hemitone). The 9:8 intervals were chosen and the gaps filled in with hemitones in order to achieve the essential harmonies of the fourth (4:3), the fifth (3:2), and the octave (2:1): 9:8 is the ratio of the fifth to the fourth. In decimals, these intervals have the ratios:**

**a hemitone = 256/243 ≈ 1.0535.**
**a second = 9/8 = 1.125**

**The Pythagorean scale is the pattern of tones based upon intervals: two seconds, hemitone, three seconds, hemitone:**

| Tonic:1:1 | 9:8 | 81:64 | 4:3 | 3:2 | 27:16 | 243:128 | 2:1 |
|---|---|---|---|---|---|---|---|
| Ratio: | 9:8 | 9:8 | 256:243 | 9:8 | 9:8 | 9:8 | 256:243 |

**Rendering the tonic in decimals is:**

| Tonic:1.0 | 1.125 | 1.2656 | 1.3333 | 1.5 | 1.6875 | 1.8984 | 2.0 |
|---|---|---|---|---|---|---|---|

**The Pythagorean scale contains four fifths and five fourths, more commensurables than can be attained from any other eight notes (Jeans, J.; "Science and Music". New York: Dover, pp. 166-172, 1968).**

**Compare the Pythagorean scale with the equi-tempered scale to which a modern piano or guitar is tuned. It consists of 12 equi-spaced pitches, where each of the 11 intervals is called a half-step. Two half-steps are called a whole-step. These intervals have ratios:**

**half-step = $12^{th}$-root(2) = $\sqrt[12]{(2)}$ ≈ 1.05946 (more than a hemitone)**
**whole-step = $6^{th}$-root(2) = $\sqrt[6]{(2)}$ ≈ 1.12246 (less than a second)**

**The equi-tempered major scale is the pattern of tones based upon intervals: two whole-steps, half-step, three whole-steps, half-step; which, rendered in decimals, is:**

| Tonic:1.0 | 1.1225 | 1.2599 | 1.3348 | 1.4983 | 1.6818 | 1.8877 | 2.0 |
|---|---|---|---|---|---|---|---|

**It's close to the Pythagorean tonics, but notice the differences:**

| Tonic:1.0 | 1.125 | 1.2656 | 1.3333 | 1.5 | 1.6875 | 1.8984 | 2.0 |
|---|---|---|---|---|---|---|---|
| Tonic:1.0 | 1.1225 | 1.2599 | 1.3348 | 1.4983 | 1.6818 | 1.8877 | 2.0 |
| Diff's: 0.0 | -.0025 | -.0057 | +.0015 | -.0017 | -.0057 | -.0107 | 0.0 |

**Ti is the most out of tune, with Mi and La about half as bad. The important Fa (fourth) and So (fifth) are quite close to the Pythagorean pitches.**