Review (1 – 7)

R.1 **Numbers and Operations (+, –, ×, ÷, ^)**
Laws of Algebra and Precedence Rules
Divide is multiply by reciprocal; **PEMDAS**
Evaluation, Substitution, Simplification, Expanding, Factoring
Expanding uses Distributive Law; Factoring is reverse Expanding

R.2 **Solving Linear Equations in One Variable**
"Both Sides" **(+, –, ×, ÷, ^)** until variable alone = number
"Both Sides" to clear denominators
Solving Linear Inequalities in One Variable
Solve equation and test points
Word Problems: Find Model with unknowns as variables;
Substitute knowns yielding equation(s); Solve.
Check solution: arithmetic and common sense.

R.3 **Cartesian Coordinates for the Plane - (x,y)**
Graph Linear Equations in Two variables - graph is a line
Ax + By = C : find X-Int (y=0) and Y-Int (x=0)
y = mx + b : b = Y-Int, m = slope = $^{rise}/_{run}$ = $^{up}/_{over}$ = $^{\Delta y}/_{\Delta x}$
For any two points on the line, **(x_1,y_1) & (x_2,y_2):**
$m = {}^{(y2 - y1)}/_{(x2 - x1)}$ **$b = y_1 - mx_1$**

R.4 **Systems of Linear Equations – Substitute variables or Add equations**

R.5 **Polynomials** (Degree, Monomial, Binomial, Term)
Operations (+, –, ×, ÷) : Manipulate like decimal whole numbers
Long division yields a remainder
FOIL – Sum of 4 pairs of products - First, Outer, Inner, Last

R.6 **Factoring: "Primes", "Pull Out", "by Grouping",**
Reverse FOIL: **$(x + p)(x + q) = x^2 + (p+q)x + pq$.**
Standard Factor Rules **$A^2 - B^2 = (A + B)(A - B)$**
$(A + B)^2 = A^2 + 2AB + B^2$
$(A - B)^2 = A^2 - 2AB + B^2$
Polynomial Equations: Solve by "Zero Product Split":
Polynomial on one side; **0** on other side; Factor and split.

R.7 **Rational Expressions: Quotient of Polynomials**
Domain is where denominator is NOT zero
Operations (+, –, ×, ÷) : Manipulate like decimal fractions
Compound Expressions: Eliminate multiple levels of denominators
Rational equations: Solve by eliminating denominators, thus
Transforming the equation into a polynomial equation (see above).

<u>Functions (3, 8, 12)</u>

Function: <u>Well-defined</u> rule, **$y = f(x)$,** relating a **domain** set to a **range** set.

domain = { x = input **= independent** variable **}**

range = **{ y =** output = **dependent** variable **}**

Graphing Functions: (see below for TI-Calculator):

Plot points **$(x, y=f(x))$** for several values (build table) and connect points

Find **x-intercepts** (where **$f(x)=0$**) and **y-intercepts ($y = f(0)$)**

Graph defines a function if graph satisfies **Vertical Line Rule**

Finding Domains : {**x** where formula works} : Points on **x-axis** above/below the graph.

Remember: One cannot divide by **0** or take square (even) roots of **"< 0"**

Finding Ranges : {**y** which come out} : Points on **y-axis** to left/right of graph.

Linear Functions ($y=mx+b$): Graph via **b** (where to start) and **m** (how to move).

Linear Reciprocate Functions: "blows up" when denominator = 0; "flattens" for big x.

Absolute Value Function: "V"=shaped

Algebra: Given two functions**, $y = f(x)$** and **$y = g(x)$,** define 4 new functions:

$(f+g)(x) = f(x) + g(x)$ **$(f-g)(x) = f(x) - g(x)$**

$(f\cdot g)(x) = f(x) \cdot g(x)$ **$(f\div g)(x) = f(x) \div g(x)$**

Composition of functions: Given two functions, **$y = f(x)$** and **$y = g(x)$,**

define the composition function: **$f \circ g(x) = f(g(x))$.** (The order matters.)

1-1 Functions: $y = f(x)$ is **1-1 if $x_1 \neq x_2$ means $f(x_1) \neq f(x_2)$.**

The graph of a **1-1** function satisfies the **Horizontal Line Rule**

Inverse Functions: if **$y = f(x)$** is **1-1,** there is an inverse, **$y = f^{-1}(x)$** satisfying:

$f \circ f^{-1}(x) = x = f^{-1} \circ f(x)$

Finding Inverse: Start: **$y = f(x)$**; solve for **x** as **$x = f^{-1}(x)$;** swap **x** and **y.**

The graph of **$f^{-1}(x)$** is the graph of **$f(x)$** reflected across the line **$y = x$.**

TI-83/84 Graphing Keys

"X,T,θ,n" specifies the independent variable

"Y=" specifies function **"Graph"** draws graph

"Window" specifies ticky marks **"Zoom-6"** for standard ticky marks

"2nd-Window" for table start **"2nd-Graph"** displays table

"Trace" to display **"bug"**, use **"left"** & **"right"** arrows to move bug (If more than one equation is graphed, use the **"up"** & **"down"** arrows to switch between graphs.)

"2nd-Trace" to find various tasks:

Value (specify **X**) **Zero** (where **Y=0**)

Min & **Max** (for **Y**): prompts for left & right bounds and a guess.

Intercept: prompts for which two "curves" and a guess.

Inequalities (9)

Linear Inequalities: Solve via "Both Sides" (remember the flip).
Solutions are sets: graph sets and write in set-builder or interval notation.

Business Models: **x = # units; F = $Fixed costs, c = $cost/unit, p = $price/unit,**
$Cost = C(x) = F + cx; $Revenue = R(x) = px; Breakeven when **R(x) = C(x).**

Compound inequalities: Solve each and combine the solution sets using:
Union (OR) = $\cup$ (merge) **Intersection (AND) =** $\cap$ (in commom)
(Alternatively, Solve each equation, plot points, check regions.)

Absolute value Equations: Split into two cases**(±),** solve, check.
Absolute value Inequalities: Solve equation, plot points, check regions.

Graphing Inequalities in Two Variables
Graph the corresponding linear equations getting lines;
Check points to determine regions (which side of line).

Radicals (10)

Roots are inverse operation of raising to a power.

$\sqrt{x^2} = |x| = (\sqrt{x})^2$ $\sqrt[3]{x^3} = x = (\sqrt[3]{x})^3$

$\sqrt[4]{x^4} = |x| = (\sqrt[4]{x})^4$ $\sqrt[5]{x^5} = x = (\sqrt[5]{x})^5$

$\sqrt[n]{x \cdot y} = \sqrt[n]{x} \cdot \sqrt[n]{y}$ **and same for division**

Radicals and Fractional Exponents are equivalent: (use Laws of Exponents)

$x^{-n} = 1/x^n$ $\sqrt[n]{x} = x^{1/n}$ $\sqrt[n]{(x^m)} = (\sqrt[n]{x})^m = x^{m/n}$

Simplify Radicals : (i) as few radicals as possible
(ii) pull as much outside the radical as possible
(iii) no radicals in denominators

(+,–) like radicals: $\sqrt{20} + \sqrt{45} = \sqrt{4 \cdot 5} + \sqrt{9 \cdot 5} = 2\sqrt{5} + 3\sqrt{5} = 5\sqrt{5}$

Multiply sums with **FOIL**: $(\sqrt{x} + 4)(\sqrt{x} - 2) = x + 2\sqrt{x} - 8$

Use conjugates, $(\sqrt{x} + 3)(\sqrt{x} - 3) = x - 9$, to clear denominators

Radical functions - must restrict the domain for even roots.

$\sqrt[even]{\geq 0} = \geq 0$ $\sqrt[odd]{\pm} = \pm$

Solving equations - raise both sides to a power to remove radical (and always **check**).

Complex Numbers - define: $i = \sqrt{-1}$ **, imaginary unit**, $i^2 = -1$.
Numbers like **a + bi**, are called complex numbers. Use Laws of Algebra.
To divide, multiply top/bottom by complex conjugate of denominator:
Complex conjugates: $(a + bi)(a - bi) = a^2 + b^2$

Quadratics (11)

Quadratic Equations: $ax^2 + bx + c = 0$

Solve by **Quadratic Formula**: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (complete the square)

Quadratic Functions: $y = f(x) = ax^2 + bx + c = a(x - h)^2 + k$ $(a \neq 0)$

Vertex of parabola: $x = -b/2a = h$ and $y = f(-b/2a) = k =$ **max/min**

Shape determined by **|a|: steep** for **|a| big**, **flat** for **|a| small.**

For a > 0: parabola is concave **up** vertex is a **minimum.**

For a < 0: parabola is concave **down** vertex is a **maximum.**

Domain = **{ all x }**. Range = **{ y ≤ max** or **y ≥ min }**.

Y-intercept: $y = f(0) = c$.

X-intercept: Solve quadratic equation: $0 = f(x)$

Applications:

Distance from (x_1,y_1) to $(x_2,y_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint between (x_1,y_1) and $(x_2,y_2) = \left({}^{(x2 + x1)}/_2 , {}^{(y2 + y1)}/_2 \right)$

Maximum and **Minimum** problems – find vertex

Exponentials and Logarithms (12)

Exponential and Logarithmic Functions (recall laws of exponents)

$y = f(x) = Ca^x$ is exponential function for $a > 0$ $(\neq 1)$ $C > 0$.

Domain = **{ all x }** Range = **{ y > 0 }**

$y = \log_a(x)$ is logarithmic function for $a > 1$.

$y = \log(x) = \log_{10}(x)$ is common logarithmic function.

$y = \ln(x) = \log_e(x)$ is natural logarithmic function **(e is natural base).**

Domain = **{ x > 0 }** Range = **{ all y }**

Inverse Functions: $\log_a(a^x) = x = a^{\log a(x)}$

$\log_a(1) = 0$ $\log_a(a) = 1$

Calculator: $\log_a(x) = \log(x) / \log(a) = \ln(x) / \ln(a)$

Balances, B(t), as a function of time, **t,** in years, **B(0)** starting balance:

earning r% interest compounded once or **m-times** per year**:**

$B(t) = B(0)(1 + r)^t$ $B(t) = B(0)(1 + r/m)^{mt}$

earning r% interest compounded continuously grow as**:**

$B(t) = B(0)e^{tr}$ **(e is natural base)**

Amounts, A(t), growing or decaying exponentially as a function of time, **t:**

Doubling time (D): $A(t) = A(0)2^{t/D}$

Half-Life time (H): $A(t) = A(0)(1/2)^{t/H} = A(0)2^{-t/H}$

Richter Scale for Earthquakes and pH are logarithmic functions.