Math012 - Final Exam

1.(14) Evaluate (answer is an integer or simple fraction):

$2 \times (3 + 2 \wedge 3) - 5 \wedge 2 + 5 \times 4 = 2 \times (11) - 25 + 20 = 17$

$(2^3)^2 = (2\wedge 3)\wedge 2 = 8\wedge 2 = 64$ $\qquad$ $2^{(3\wedge 2)} = 2\wedge(3\wedge 2) = 2\wedge 9 = 512$

$\sqrt[3]{-27} = -3$ $\qquad$ $\sqrt{0.0064} = .08$

$\dfrac{\sqrt{48} - \sqrt{12}}{\sqrt{3}} = \dfrac{4\sqrt{3} - 2\sqrt{3}}{\sqrt{3}} = 2$ $\qquad$ $\dfrac{\sqrt{98}}{\sqrt{50}} = \dfrac{7\sqrt{2}}{5\sqrt{3}} = \dfrac{7}{5}$

$\sqrt{13 \times 2 \wedge 2 - 4 \times 3 - 4} = \sqrt{52 - 12 - 4} = 6$

$\log_2(8) = \log_2(2^3) = 3$ $\qquad$ $\log_5(\sqrt{5}) = \log_5(5^{0.5}) = 0.5$

$\log(0.000001) = \log_{10}(10^{-6}) = -6$ $\qquad$ $\log(100{,}000) = \log_{10}(10^5) = 5$

$\log_b(b) = 1$ $\qquad$ $\log_b(1) = 0$

2.a(4) Simplify each to one exponent per variable (no radicals):

$x^{1/3}x^{4/5} = x^{17/15}$ 1/3 + 4/5 = 17/15

$$\frac{(x^3y^2z^4)^2}{(x^2y^3z^2)^3} = \frac{x^6y^4z^8}{x^6y^9z^6} = \frac{z^2}{y^5} = z^2\,y^{-5}$$

2.b(4) Write each with one radical and with no denominators containing radicals and no denominators inside a radical:

$$\frac{\sqrt{x+2}}{\sqrt{x+3}}\,\frac{\sqrt{x+3}}{\sqrt{x+3}} = \frac{\sqrt{(x+2)(x+3)}}{x+3}$$

$$\frac{\sqrt{x}+2}{\sqrt{x}+3}\,\frac{\sqrt{x}-3}{\sqrt{x}-3} = \frac{x-\sqrt{x}-6}{x-9}$$

2.c(6) Write each as a single, simplified rational expression:

$$\frac{x^2+x-12}{x^2+2x-8} = \frac{(x+4)(x-3)}{(x+4)(x-2)} = \frac{x-3}{x-2}$$

$$\frac{x+3}{x+1} - \frac{x-4}{x+2} = \frac{(x+3)(x+2) - (x-4)(x+1)}{(x+1)(x+2)}$$

$$= \frac{(x^2+5x+6) - (x^2-3x-4)}{(x+1)(x+2)} = \frac{8x+10}{x^2+3x+2}$$

$$\frac{\frac{x+3}{x+1}}{\frac{x-4}{x+2}} = \frac{(x+3)}{(x+1)}\,\frac{(x+2)}{(x-4)} = \frac{x^2+5x+6}{x^2-3x-4}$$

3.(12) Graph the linear function: $y = f(x) = 2x - 3$.

What is the equation of its inverse function, $f^{-1}(x)$?

Since f(x) does: 1st) times 2 ; and 2nd) minus 3.

Then $f^{-1}(x)$ does: 1st) plus 3 ; and 2nd) divide by 2.

$$y = f^{-1}(x) = \frac{x + 3}{2} = (1/2)x + 3/2$$

Graph the inverse function.

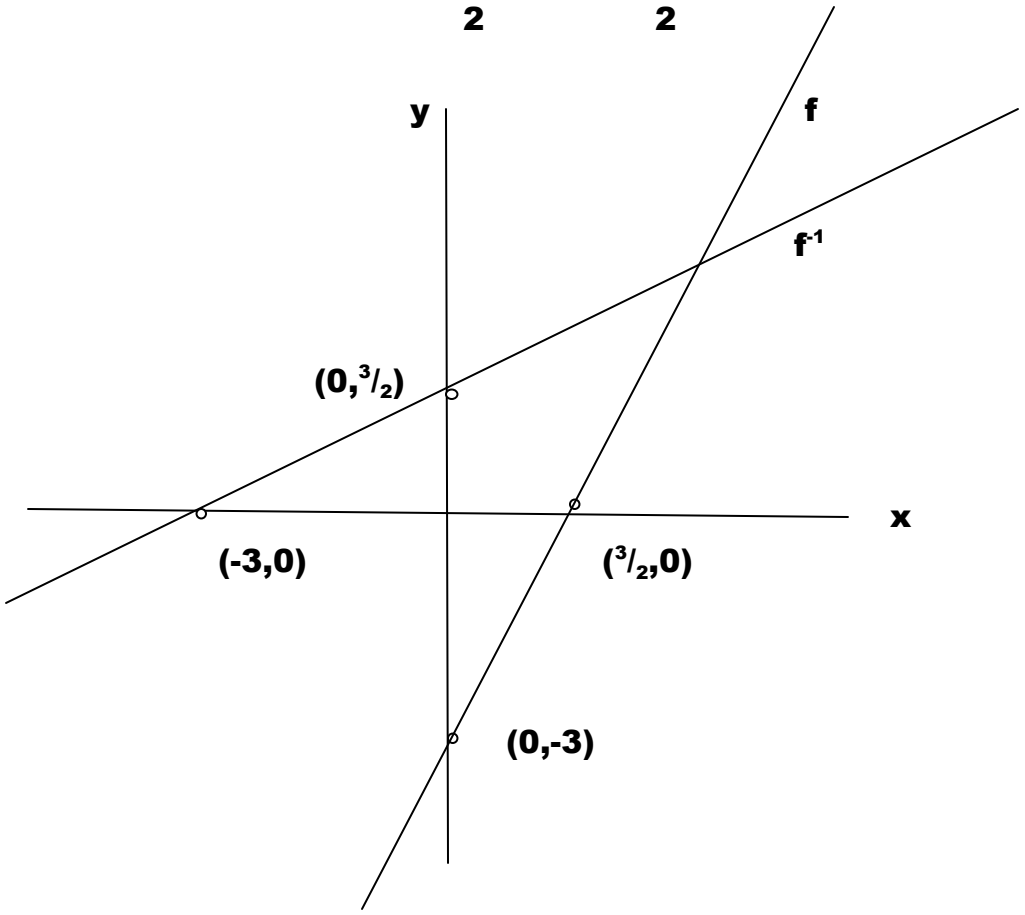
f(x) has: m = 2 and b = -3; $f^{-1}(x)$ has: $m = 1/2$ and $b = 3/2$

Explicitly compute the compositions (Hint: they should be equal):

$f \circ f^{-1}(x)$ and $f^{-1} \circ f(x)$.

$$f \circ f^{-1}(x) = f\left(\frac{x+3}{2}\right) = 2\left(\frac{x+3}{2}\right) - 3 = x + 3 - 3 = x$$

$$f^{-1} \circ f(x) = f^{-1}(2x - 3) = \frac{(2x - 3) + 3}{2} = \frac{2x}{2} = x$$



4.(5) Complete the second table to define the inverse function for the function defined on the first 10 whole numbers by the first table:

x	f(x)
0	1
1	4
2	9
3	6
4	2
5	7
6	8
7	3
8	5
9	0

x	$f^{-1}(x)$
0	9
1	0
2	4
3	7
4	1
5	8
6	3
7	5
8	6
9	2

5.(10) Solve and check the following (quadratic) equations.

a) Solve: $2(x-3)^2 = 16$

$(x-3)^2 = 8$

$x - 3 = \pm\sqrt{8}$

$x = 3 \pm\sqrt{8}$

b) Solve: $x^2 + x = 5$

$x^2 + x - 5 = 0$

Quadratic Formula:

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-5)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{21}}{2}$$

6.(7) Solve and check the following: $|2x + 1| = 3$

$2x + 1 = 3$	or	$2x + 1 = -3$
$2x = 2$		$2x = -4$
$x = 1$		$x = -2$
check		check

7.(8) Solve and check the following:

a) $2x + 5 > 3$ and $3x - 1 < 8$

$2x > -2$ and $3x < 9$

$x > -1$ and $x < 3$

$\{ x \mid -1 < x < 3 \} = (-1 , 3)$

b) $2x + 5 < 3$ or $3x - 1 > 8$

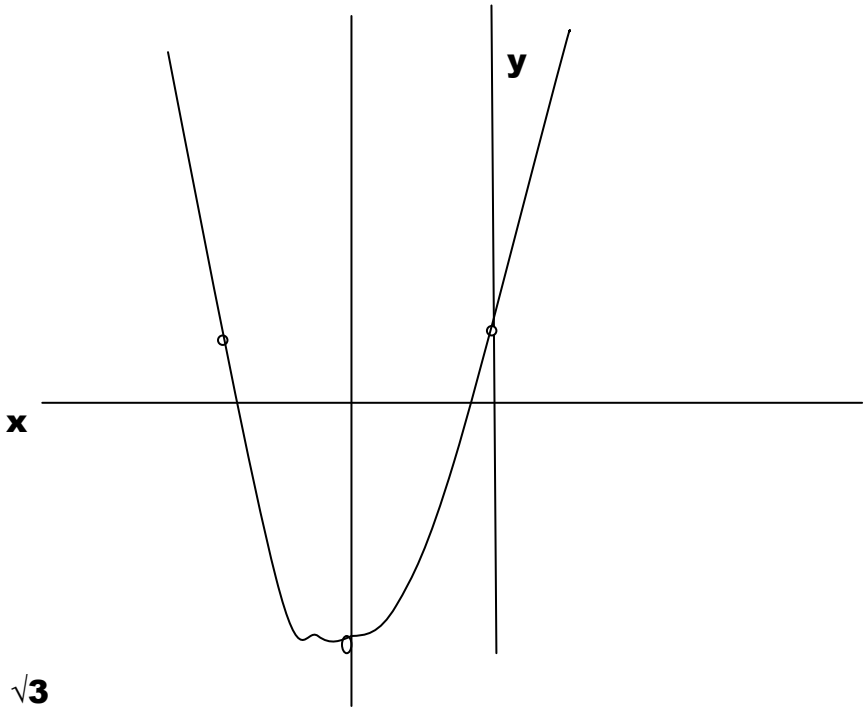
$2x < -2$ or $3x > 9$

$x < -1$ or $x > 3$

$\{ x \mid x < -1 \text{ or } x > 3 \} = (-\infty , -1) \cup (3 , \infty)$

8.(10) Consider the quadratic function: $y = f(x) = x^2 + 4x + 1$. Complete the table, sketch its graph and specify the 4 facts below. Draw a dotted vertical line for the axis of symmetry.

x	y = f(x)
-3	9 – 12 + 1 = -2
-2	4 – 8 + 1 = -3
-1	1 – 4 + 1 = -2
0	1
1	1 + 4 + 1 = 6



Vertex: (-2,-3)

Y-Int: (0,1)

X-ints: Q.F.

$$x = \frac{-4 \pm \sqrt{16 - 4(1)1}}{2(1)} = -2 \pm \sqrt{3}$$

Or $x \approx -3.7, -0.3$

9.(10) Consider the two points: (-2,-3) and (6,3).

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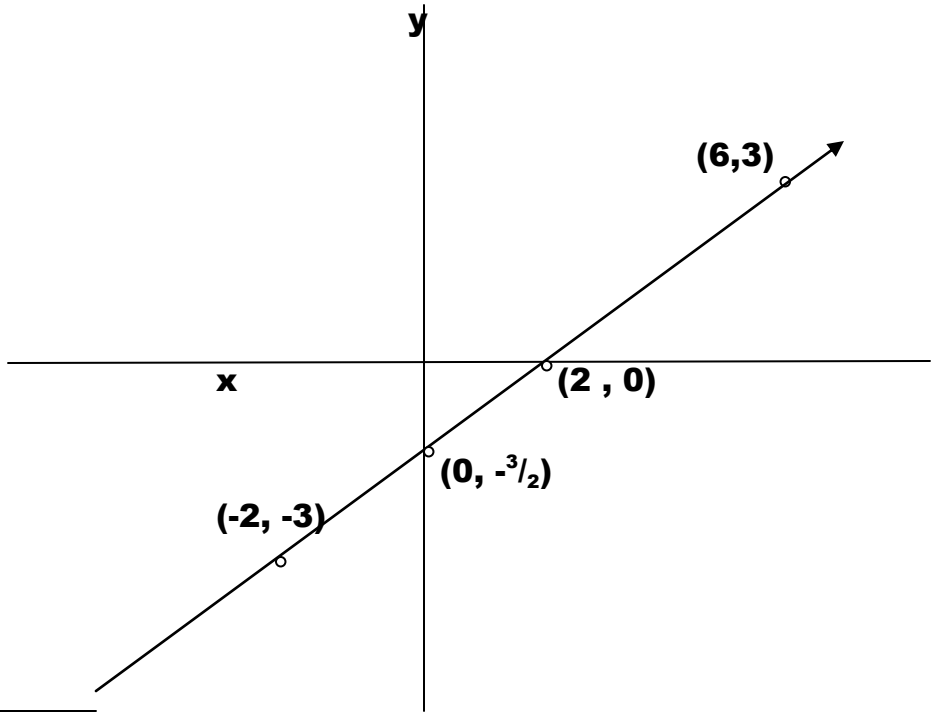
a) Graph and find the equation of the line which joins them.

b) Find the distance between them.

c) Find the midpoint between them.

d) Label both the X- & Y-intersects.



$$m = \frac{3 - -3}{6 - -2}$$

$$m = {}^{6}/_{8}$$

$$y = ({}^{3}/_{4})x + b$$

$$3 = ({}^{3}/_{4})6 + b$$

$$b = -{}^{3}/_{2}$$

$$dist = \sqrt{(6 - -2)^2 + (3 - -3)^2}$$

$$= 10$$

$$MP = \left(\frac{6 + -2}{2} , \frac{3 + -3}{2} \right)$$

$$= (2 , 0)$$

10.(5) You plan to make and sell doodads. You estimate your:

i) monthly fixed costs are $1800 to rent a shop;
ii) variable costs are $11/doodad (materials and power);
ii) target price to sell is $17/doodad.

a) Write functions for monthly revenue and cost in terms of x = number of doodads made and sold per month.

R(x) = 17x **C(x) = 11x + 1800**

b) What is your break-even point in doodads sold per month?

17x = 11x + 1800
6x = 1800
x = 300 doodads.

c) The break-even point in b) is too high. What price per doodad must you charge to breakeven at 200 doodads sold per month?

Let p = new price per unit and x = 200.

200p = 11(200) + 1800 = 4000 ; so p = $20/doodad.

12.(5) For an F-250 truck, gas mileage (m) depends upon velocity (v) according to the quadratic function:

$$m(v) = 0.486v - 0.0054v^2$$ **; with v in mph and m in mpg.**

At what velocity v does the truck get the maximum gas mileage? What is the maximum gas mileage?

The max is at the vertex, which is half-way between X-ints.

0 = 0.0054v(90 − v) ; so X-int's at v = 0, 90 ; so vertex at:

So, v = 45mph ; m(45) = 10.9mpg