Math151 - Practice Test on Chapters 1 and 2

1.

1.a. Simplify: (at most one radical)

$$(4 + 2^3) \bullet \sqrt{7} + 3^2 - 12 \div 4 = 12 \bullet \sqrt{7} + 9 - 3 = 6 + 12\sqrt{7}$$

$$3\sqrt{20} + 2\sqrt{45} = 3\sqrt{4 \cdot 5} + 2\sqrt{9 \cdot 5} = 3 \cdot 2\sqrt{5} + 2 \cdot 3\sqrt{5} = 12\sqrt{5}$$

$$(4 + 2)^3 \bullet (\sqrt{7 + 3^2} - 12) \div 4 = 216 \bullet (\sqrt{16} - 12) \div 4 = -432$$

$$\frac{3 + 2\sqrt{5}}{2 - \sqrt{5}} \cdot \frac{2 + \sqrt{5}}{2 + \sqrt{5}} = \frac{6 + 3\sqrt{5} + 4\sqrt{5} + 10}{4 - 5} = -16 - 7\sqrt{5}$$

1.b. Simplify: (simple fraction)

$$\frac{\frac{a}{b} - \frac{b}{a}}{\frac{a^2 - 2ab + b^2}{a}} = \frac{\frac{a^2 - b^2}{ab}}{\frac{(a-b)^2}{a}} = \frac{a(a-b)(a+b)}{ab(a-b)^2} = \frac{(a+b)}{b(a-b)}$$

$$\frac{x+3}{x-2} - \frac{x-1}{x+4} = \frac{(x+3)(x+4) - (x-1)(x-2)}{(x-2)(x+4)} = \frac{(x^2+7x+12) - (x^2-3x+2)}{x^2 + 2x - 8}$$

$$= \frac{10x + 10}{x^2 + 2x - 8}$$

1.c. Simplify (single exponent per variable, no radicals, no division):

$$\sqrt{a^3} \times \sqrt[3]{a^2} = a^{3/2} \cdot a^{2/3} = a^{3/2 + 2/3} = a^{13/6}$$

$$\frac{b\sqrt{b^3a^5}}{b^3} = \frac{(b^3a^5)^{1/2}}{b^2} = b^{3/2}\, a^{5/2}\, b^{-2} = b^{-1/2}a^{5/2}$$

1.d. Simplify (hint: factor) the following:

$$(x-1)(x^2-4)+(x^2-1)(x+2) = (x-1)(x-2)(x+2)+(x-1)(x+1)(x+2)$$
$$= (x-1)(x+2)\left((x-2)+(x+1)\right) = (x-1)(x+2)(2x-1)$$

$$a(b-c) - 2a^2(c-b) = a(b-c) + 2a^2(-c+b) = a(b-c)(1+2a)$$

1.e. Simplify the following (at most one radical):

$$(\sqrt{x}+2)(2\sqrt{x}-4) = 2x - 4\sqrt{x} + 4\sqrt{x} - 8 = 2x - 8$$

$$\frac{x+1}{\sqrt{x+2}} + \frac{\sqrt{x+2}}{x^2-1} = (\sqrt{x+2})\left(\frac{x+1}{x+2} + \frac{1}{x^2-1}\right)$$

$$= \frac{\sqrt{x+2}\,(x^3+x^2+1)}{x^3+2x^2-x-2}$$

1.f. Simplify the following (variable appears once with one exponent):

$$\frac{(x^2yz^3)^3}{(x^4y^2z^3)^2} = \frac{x^6y^3z^9}{x^8y^4z^6} = \frac{z^3}{x^2y}$$

$$\frac{x^2y^{-5}z^3}{(x^{-4}y^2z^{-3}} = \frac{x^6z^6}{y^7}$$

2.a. Solve the following linear equation: $2x + 3 = 3x - 2$

(Both sides: + 2 & -2x) $5 = x$

2.b. Solve the following linear equation: $|2x+3| = |3x-2|$

Split into two cases

$2x + 3 = 3x - 2$ **OR** $2x + 3 = -(3x-2) = -3x + 2$

$x = 5$ $5x = -1$

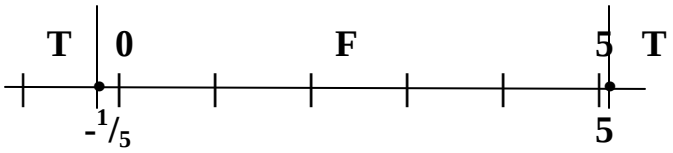
$x = -1/5$

2.c. Solve the following linear inequality: $|2x + 3| < |3x - 2|$

(Solve equation and plot points on the number line)

Check points in 3 regions: {x < -1/5} ; {-1/5 < x < 5}; {x > 5}

Check x = -1: 1 < 5 True
Check x = 0: 3 < 2 False
Check x = 6: 15 < 16 True
{ x | x < $^{-1}/_{5}$ or x > 5 }



3.a. Solve the equation: $x^2 - 8 = x + 4$

$x^2 - x - 12 = 0 = (x - 4)(x + 3)$

So: $x - 4 = 0$ **or** $x + 3 = 0$

$x = 4$ **or** $x = -3$

or, by quadratic formula: $x = \dfrac{--1 \pm \sqrt{1 - 4 \cdot 1(-12)}}{2 \cdot 1}$

$= \dfrac{1 \pm \sqrt{49}}{2} = 4 \text{ or } -3$

3.b. Solve the equation: $2x^4 - 4 = 3x^2$

$2x^4 - 3x^2 - 4 = 0$

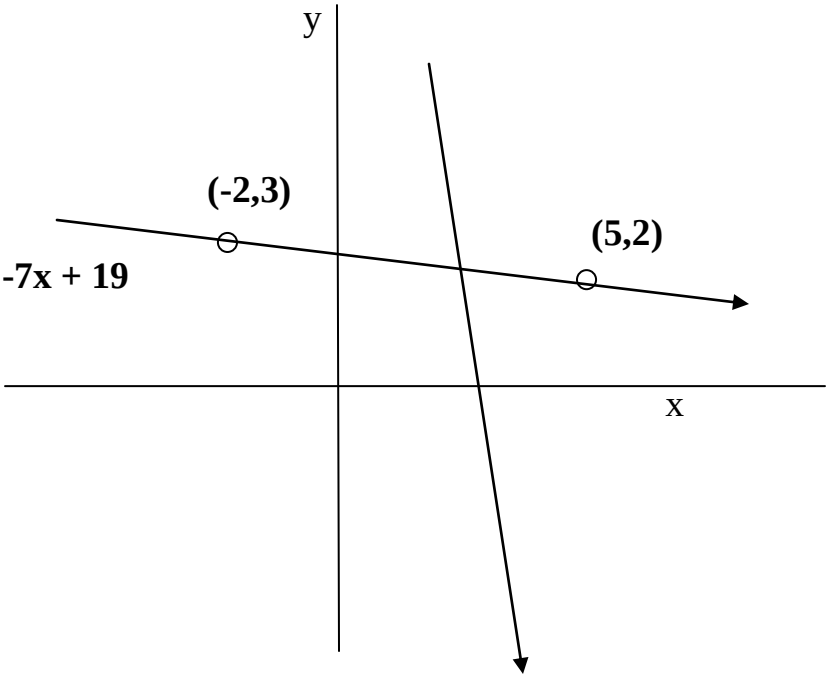
Letting $u = x^2$: $2u^2 - 3u - 4 = 0$

By quadratic formula: $u = x^2 = \dfrac{3 \pm \sqrt{9 - 4 \cdot 2(-4)}}{2 \cdot 2} = \dfrac{3 \pm \sqrt{41}}{4} \approx \dfrac{9.5}{4} \text{ or } \dfrac{-3.5}{4}$

Reject negative since $x^2 \geq 0$, and, taking square root:, $x = \pm\dfrac{\sqrt{3 + \sqrt{41}}}{2}$

4. Write the equation of the line through the two points: **(-2,3)** and **(5,2)**.
Draw its graph.
What are the slope and the X&Y-intercepts?
What is the distance between the two points: **(-2,3)** and **(5,2)**?
Interpreting the equation as a function, find the equation of its inverse.
Graph the inverse.

m = $\frac{2-3}{5-(-2)} = -\frac{1}{7}$
Y-int = $\frac{19}{7}$
X-int = 19
Equation: $y = (-\frac{1}{7})x + \frac{19}{7}$
Inverse: $y = (x - \frac{19}{7})/(-\frac{1}{7}) = -7x + 19$



Dist = $\sqrt{7^2 + 1^2} = \sqrt{50}$
MP = (1.5, 2.5)
Y-int how?
Know $y = (-\frac{1}{7})x + b$:
$2 = -\frac{5}{7} + b$
$\frac{19}{7} = b$
Equation: $y = (-\frac{1}{7})x + \frac{19}{7}$
X-int (y=0): $0 = (-\frac{1}{7})x + \frac{19}{7}$
So, x = 19

5. Consider the two functions: **$f(x) = x^2 - 1$** and **$g(x) = \sqrt{x+1}$** .

Dom(f) = {all x}; Dom(g) = {x ≥ -1}

$f+g(x) = x^2 - 1 + \sqrt{x+1}$; Dom(f+g) = {x ≥ -1}
$f \times g(x) = (x^2 - 1)(\sqrt{x+1})$; Dom(f×g) = {x ≥ -1}
$f \div g(x) = (x^2 - 1) / \sqrt{x+1}$; Dom(f÷g) = {x > -1}

$f \circ g(x) = (\sqrt{x+1})^2 - 1 = x$; Dom(f∘g) = {x ≥ -1}
$g \circ f(x) = \sqrt{(x^2 - 1) + 1} = |x|$; Dom(g∘f) = { all x}

No, f(x) is not 1-1, so they are not inverses.

6. Complete the second table to define the inverse, $\mathbf{f^{-1}(x)}$:

x	f(x)
0	7
1	2
2	5
3	6
4	1
5	9
6	0
7	8
8	4
9	3

x	$f^{-1}(x)$
0	6
1	4
2	1
3	9
4	8
5	2
6	3
7	0
8	7
9	5

8.a. Circle radius = **4** and center at **(2,-1):** $\mathbf{(x-2)^2 + (y-(-1))^2 = 4^2}$

$$\mathbf{(x-2)^2 + (y+1)^2 = 16}$$

8.b. Parabola with y-int **= 4** and vertex at **(2,-1):** $\mathbf{y = a(x-2)^2 + -1}$

But y-int is where **x = 0,** so: $\mathbf{4 = a(0-2)^2 - 1 = 4a - 1}$

$$\mathbf{5 = 4a \quad or \quad a = {}^5/_4.}$$

$$\mathbf{y = ({}^5/_4)(x-2)^2 - 1 = ({}^5/_4)x^2 - 5x + 4\ .}$$

9.a Mary can build a website in 19 hours. Jerry can build the same website in 25 hours. Working together, how long will it take the two of them to build the website?

Let t = #hrs to do one website together.
Mary works at: 1WS/19hrs = (1/19)WS/hr
Jerry works at: 1WS/25hrs = (1/25)WS/hr
Working together for t hrs to do 1 WS: $(1/19)t + (1/25)t = 1$;
combining: $44t/475 = 1$;
solving: $t = 475/44$ hrs.

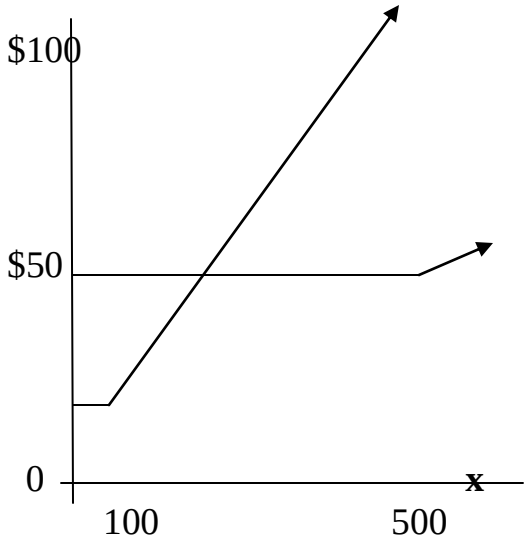
9.b You are choosing between two cell phone contracts. Poorizon costs \$19 per month plus \$0.25 per minute for any minutes over 50. Jipular costs \$49 per month plus \$0.15 per minute for any minutes over 500. Write a cost function for each. Graph both cost functions. At how many minutes per month do they cost the same?

Let **x** = #minutes

$$P(x) = \begin{cases} 19 & \text{for } x \le 50 \\ 19 + 0.25(x - 50) & \text{for } x > 50 \end{cases}$$

$$J(x) = \begin{cases} 49 & \text{for } x \le 500 \\ 49 + 0.15(x - 500) & \text{for } x > 500 \end{cases}$$

Solve: **$49 = 19 + 0.25(x - 50)$**
…… x = 170 minutes.



9.c. The function: **$d = 1.2\sqrt{h}$** , relates the distance, **d** in miles, one can see out to sea when standing a height, **h** in feet, above the water. How high must one be to be able to see **12** miles out to sea?

$d = 12 = 1.2\sqrt{h}$ or $10 = \sqrt{h}$ $100 = h$: 100ft

10. The following charts show the temperature every hour between sunrise and sunset on two given dates:

March 21			Sept 21		
Time	**Temp (°F)**		**Time**	**Temp**	
7AM	**37**		**08AM**	**51**	
8AM	**38**	**1°/hr**	**09AM**	**54**	**3°/hr**
9AM	**40**	**2°/hr**	**10AM**	**58**	**4°/hr**
10AM	**44**	**4°/hr**	**11AM**	**63**	**5°/hr**
11AM	**48**	**4°/hr**	**12PM**	**68**	**5°/hr**
12PM	**51**	**3°/hr**	**1PM**	**72**	**4°/hr**
1PM	**54**	**3°/hr**	**2PM**	**73**	**1°/hr**
2PM	**57**	**3°/hr**	**3PM**	**74**	**1°/hr**
3PM	**58**	**1°/hr**	**4PM**	**75**	**1°/hr**
4PM	**57**	**-1°/hr**	**5PM**	**75**	**0°/hr**
5PM	**56**	**-1°/hr**	**6PM**	**73**	**-2°/hr**
6PM	**53**	**-3°/hr**	**7PM**	**70**	**-3°/hr**

What was the average rate of change during each hour? Done

What was the average rate of change during each morning?

March: $\frac{51 - 37}{12 - 7} = \frac{16}{5} = 3.2°/hr$ **Sept:** $\frac{17}{5} = 3.4°/hr$

What was the average rate of change during each afternoon?

March: $\frac{53 - 51}{18 - 12} = \frac{2}{6} = 0.33°/hr$ **Sept:** $\frac{2}{7} = 0.28°/hr$

What was the average rate of change during each day?

March: $\frac{53 - 37}{18 - 7} = \frac{16}{11} = 1.45°/hr$ **Sept:** $\frac{19}{11} = 1.72°/hr$

What was the average of the hourly average rates of change?

March: 1.45°/hr **Sept: 1.73°/hr**

Comment on the differences: Rates of change over longer periods are misleading. Temperature increases fastest just before noon.