

## Math151 - Final Exam

1.(10) Evaluate exactly as a single real number (angles in radians):

a)  $\log(0.001) = -3$

b)  $\log(100,000) = 5$

c)  $\ln(e^\pi) = \pi$

d)  $\log_\pi(1) = 0$

e)  $\log_2(\sqrt{2}) = 0.5$

f)  $\sin(13\pi/6) = \sin(\pi/6) = 0.5$   
(30-60 $\Delta$ )

g)  $\sin^{-1}(1/2) = \arcsin(0.5) = \pi/6$   
“arc angle with sin=0.5”

h)  $\cos(\sin^{-1}(0.6)) = 0.8$   
“arc angle sin=0.6 is 3-4-5 $\Delta$ ”

i)  $\tan^{-1}(1) = \arctan(1) = \pi/4$   
“arc angle with tan=1”

j)  $\tan(-5\pi/6) = \tan(\pi/6) = \frac{\sqrt{3}}{3}$   
(30-60 $\Delta$ )

2.(10)

a) Simplify:  $\frac{(a^3 b^2 c^4)^2}{(a^2 b^3 c^2)^3} = \frac{a^6 b^4 c^8}{a^6 b^9 c^6} = \frac{c^2}{b^5} = c^2 b^{-5}$

b) Simplify:  ${}^5\sqrt{a^2} \times \sqrt{a^5} = a^{(2/5 + 5/2)} = a^{(29/10)} = a^{29/10}$

c) Expand into a single polynomial:  $(a^2 - 3)(3a + 1) - (3a - 2)(a^2 + 2) =$   
 $(3a^3 + a^2 - 9a - 3) - (3a^3 - 2a^2 + 6a - 4) = 3a^2 - 15a + 1$

d) Factor completely and simplify:  $2x(y^2 - x^2) - x^2(y + x) =$   
 $2x(y - x)(y + x) - x^2(y + x) = x(y + x)(2(y - x) - x) =$   
 $x(y + x)(2y - 3x)$

e) Simplify:  $\frac{1}{a+h} - \frac{1}{a} = \frac{a - (a+h)}{a(a+h)} = \frac{-h}{ha(a+h)} = \frac{-1}{a(a+h)}$

**3.(10)** Solve the following equations (solutions may be complex numbers):

**3.a**  $x^2 - 2x + 7 = 0$  (can't factor, use quadratic formula)

$$x = \frac{2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot 7}}{2 \cdot 1} = \frac{2 \pm \sqrt{-24}}{2} = 1 \pm i\sqrt{6} \quad (\text{reject non-real})$$

**3.b**  $\sqrt{2x+3} = x-2$

$$2x+3 = (x-2)^2 = x^2 - 4x + 4$$

$$0 = x^2 - 6x + 1 \quad (\text{can't factor, use quadratic formula})$$

$$x = \frac{6 \pm \sqrt{6^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{6 \pm \sqrt{32}}{2} = 3 \pm \sqrt{8} \quad (\text{reject } -) = 3 + \sqrt{8}$$

**3.c**  $3x - 4 = \sqrt{x}$       or       $3x - \sqrt{x} - 4 = 0$   
 $(3\sqrt{x} - 4)(\sqrt{x} + 1) = 0$

So:  $\sqrt{x} = 4/3$       or       $\sqrt{x} = -1$  reject

$$x = 16/9$$

**4.(8)** Simplify the following to an expression using a single trig function.

**4.a**  $\frac{1 + \cos(2x)}{\sin(2x)} = \frac{1 + 2 \cos^2 x - 1}{2 \sin x \cos x} = \frac{\cos x}{\sin x} = \cot x$

**4.b**  $\frac{\sec(x) - \cos(x)}{\tan(x)} = \frac{\frac{1}{\cos x} - \frac{\cos x}{1}}{\frac{\sin x}{\cos x}} = \frac{1 - \cos^2 x}{\sin x} = \sin x$

5.(7) Graph and find the equation of the line thru the points:  
 (-1,-2) and (4,2) .

What is the **slope**?  $m = \frac{2 - (-2)}{4 - (-1)} = \frac{4}{5}$

What is the exact **y-intercept**?  $(0, -1.2) = (0, -\frac{6}{5})$

What is the exact **x-intercept**?  $(1.5, 0) = (\frac{3}{2}, 0)$

What is the **distance** between the two points?  $= \sqrt{4^2 + 5^2} = \sqrt{41}$

What is the **midpoint** between the two points?

$= (\frac{-1 + 4}{2}, \frac{-2 + 2}{2}) = (1.5, 0)$

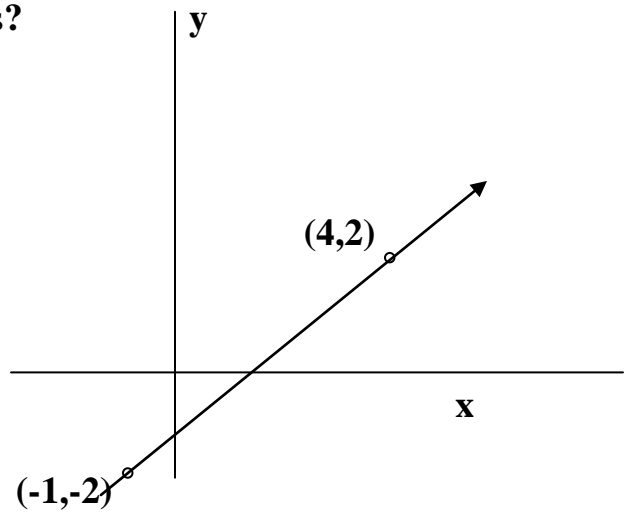
$y = (\frac{4}{5})x + b$  (to find b)

$2 = (\frac{4}{5})4 + b$

$2 - \frac{16}{5} = b$

$-\frac{6}{5} = b$

$y = (\frac{4}{5})x - \frac{6}{5}$



6.(5) Sketch the graph and find the equation of a **third** degree polynomial function and with:

**x-intercepts** (roots) at  $x = -3$ ,  $x = -1$  and  $x = 2$ ;

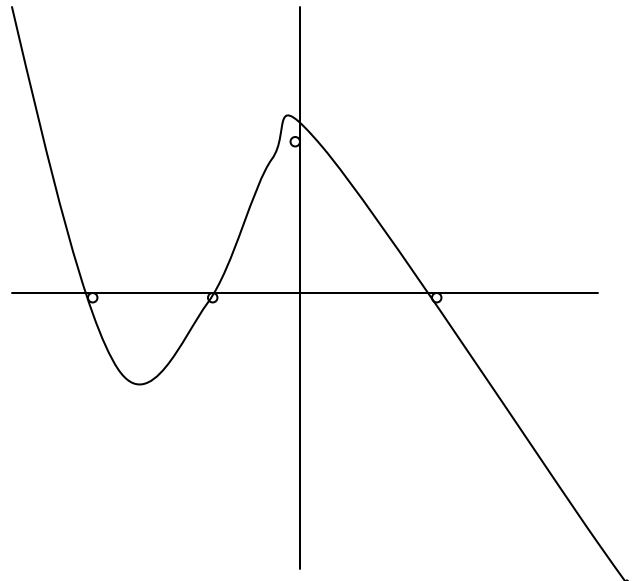
**y-intercept** at  $y = 2$ .

$y = a(x+3)(x+1)(x-2)$

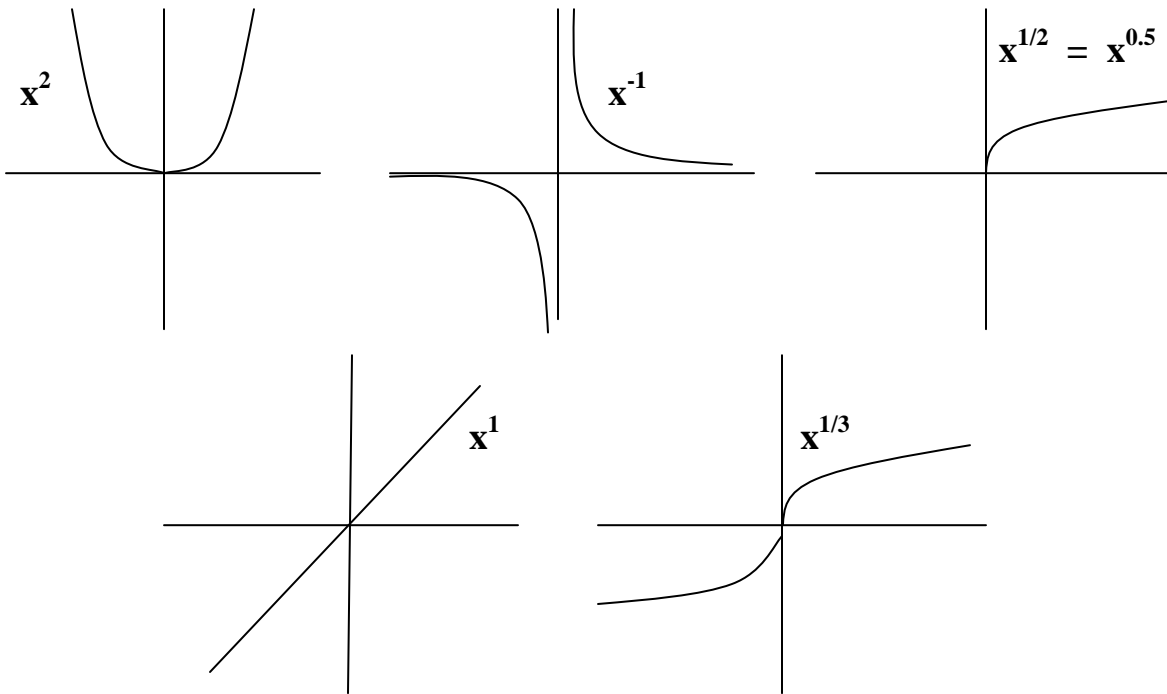
$= a(x^3 + 2x^2 - 5x - 6)$

when  $x = 0$ ,  $y = -6a = 2$

so,  $a = -1/3$  .



7.(5) Label each graph as a power function,  $y = x^a$ , for some  $a$ .



8.(10) Analyze the rational function:  $y = f(x) = \frac{2x}{x^2 - 4}$ .

**Complete** the table and **plot** those points.

**Sketch** the graph.

What is the **domain**?  $\{x \neq \pm 2\}$

What is the **range**?  $\{ \text{all } y \}$

What is the **y-intercept**?  $y = 0$

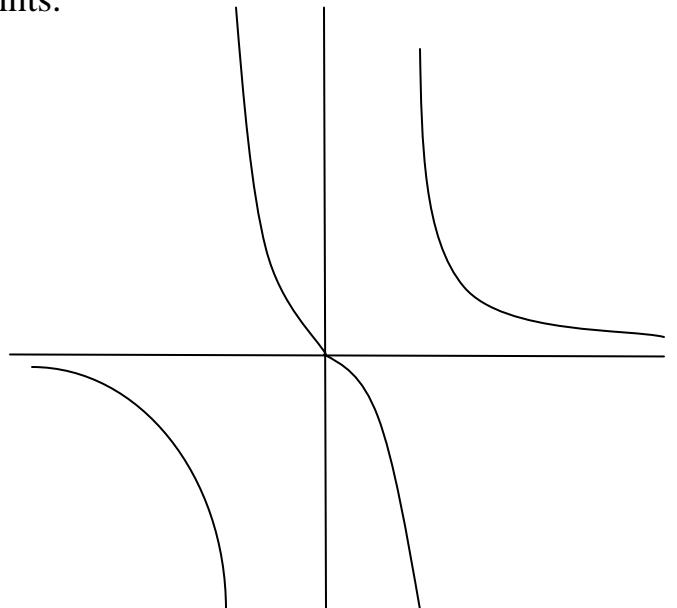
What are the **x-intercepts**?  $x = 0$

What are the **asymptotes**?

**VA:  $x = \pm 2$  ; HA:  $y = 0$**

<u>x</u>	<u>y</u>
-3	-6/5
-1	2/3
0	0
1	-2/3
3	6/5

**Odd Symmetry**



9.(5) Analyze the function:  $y = f(x) = 2 + \log_2(x + 2)$

(Complete a table and plot points if it helps)

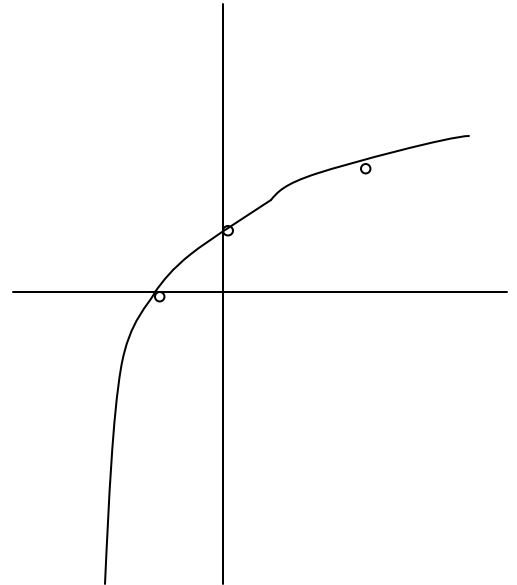
**Sketch** the graph.  $\{x > -2\}$   $\{\text{all } y\}$

What is the exact **y-intercept**?  $y = 3$

What is the exact **x-intercept**?  $x = -7/4$

What is the **asymptote**?  $x = -2$

<u>x</u>	<u>y</u>
-2	err
-1	2
0	3
2	4
6	5



10.(5) Analyze the function:  $y = f(x) = 2^{x+1} - 1$

(Complete a table and plot points if it helps)

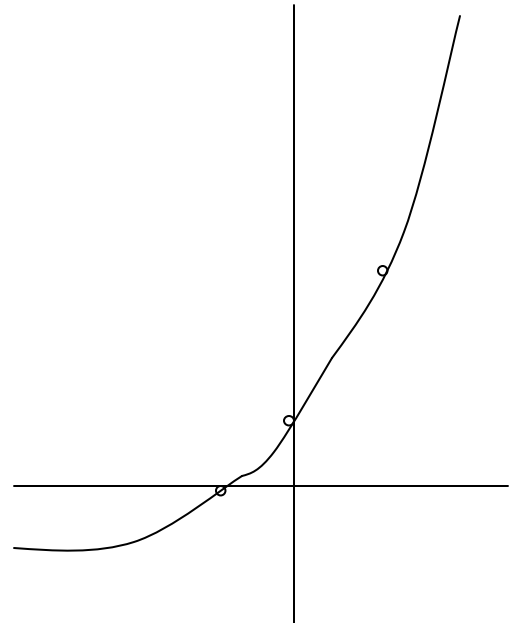
**Sketch** the graph.  $\{\text{all } x\}$   $\{y > -1\}$

What is the exact **y-intercept**?  $y = 1$

What is the exact **x-intercept**?  $x = -1$

What is the **asymptote**?  $y = -1$

<u>x</u>	<u>y</u>
-3	-3/4
-1	0
0	1
1	3
2	7



11.(7) Analyze the function:  $y = f(x) = 2\sin(x - \pi/6)$

**Complete** the table and **plot** those points.

**Sketch** the graph.

What is the **domain**?  $\{\text{all } x\}$

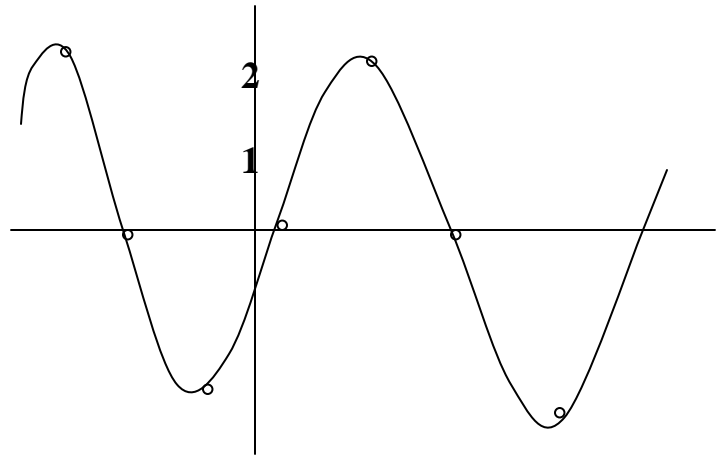
What is the **range**?  $\{-2 \leq y \leq 2\}$

What is the **period**?  $2\pi$

What is the **y-intercept**?  $y = -1$

What is one **x-intercept**?  $x = \pi/6$   
 $(\pm n\pi)$

<u>x</u>	<u>y</u>
$-\pi/3$	$-2$
$-\pi/6$	$-\sqrt{3}$
$0$	$-1$
$\pi/6$	$0$
$\pi/3$	$1$



12.(3) Find the equation for a cosine function of the form:

$$y = f(x) = A\cos(ax + b)$$

with amplitude = 3, period =  $\pi$ , and y-intercept =  $3/2 = 1.5$ .

$$y = 3 \cos(2x + \pi/3)$$

$$y(0) = 3\cos(\pi/3) \quad \text{check.}$$