Math151 - Practice Test on Chapters 3 and 4

0. Solve (include complex solutions):

0.a. $x^2 - 7 = 3x$

$x^2 - 3x - 7 = 0$ Can't be factored, so use quadratic formula.

$$x = \frac{3 \pm \sqrt{9 - 4\cdot 1(-7)}}{2\cdot 1} = \frac{3 \pm \sqrt{37}}{2}$$

0.b $x^2 + 7 = 3x$

$x^2 - 3x + 7 = 0$ Can't be factored, so use quadratic formula.

$$x = \frac{3 \pm \sqrt{9 - 4\cdot 1\cdot 7}}{2\cdot 1} = \frac{3 \pm \sqrt{-19}}{2} = \frac{3 \pm i\sqrt{19}}{2}$$

0.c. $x^3 + 9 = 3x^2 + 3x$

$x^3 - 3x^2 - 3x + 9 = 0$

$x^2(x - 3) - 3(x - 3) = 0$

$(x^2 - 3)(x - 3) = 0$

$(x - \sqrt{3})(x + \sqrt{3})(x - 3) = 0$

So: $x = -\sqrt{3}$, $\sqrt{3}$ or 3.

0.d $x^3 + 9 = 3x^2 - 3x$

$x^3 - 3x^2 + 3x + 9 = 0$

$x^2(x - 3) + 3(x + 3) = 0$ Can't be factored,

so graph on calculator and use 2nd-Trace-5:

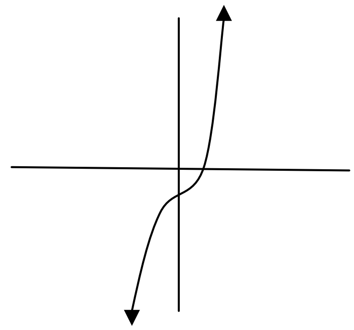
Solution: $x \approx -1.1544$

For exact solutions, including complex ones, try: www.wolframalpha.com:

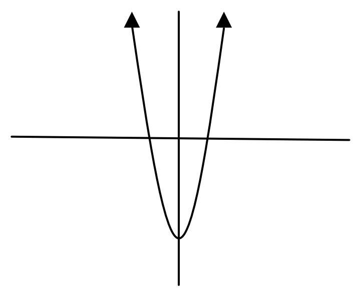
$$x = 1 - \sqrt[3]{10}\ , \qquad 1 + \frac{\sqrt[3]{5}(1 - i\sqrt{3})}{\sqrt[3]{4}}\ , \qquad 1 + \frac{\sqrt[3]{5}(1 + i\sqrt{3})}{\sqrt[3]{4}}$$

1. Sketch the graph of each of the following functions. Explicitly define the domains and ranges in set notation; explicitly label the x-intercepts and y-intercepts (if they exist); and explicitly define any asymptotes.

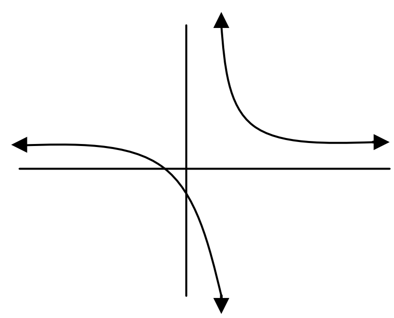
1.a. **$f(x) = x^3 - x^2 + 2x - 2 = (x - 1)(x^2 + 2)$**
Polynomial: y-int(x=0) = -2; x-int(y=0) = 1.
Domain = {all x}, Range = {all y}
Ends: $x \to \pm\infty$; $y \to \pm\infty$.



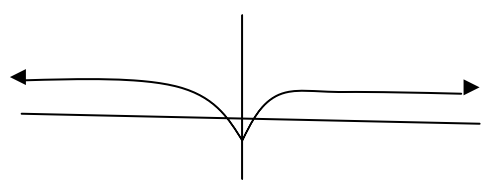
1.b. **$f(x) = x^4 + 2x^2 - 3 = (x^2 + 3)(x^2 - 1)$**
Polynomial: y-int = -3; x-int = ±1.
Dom = {all x}, Range = { $y \geq -3$}
Ends : $x \to \pm\infty$; $y \to \infty$.



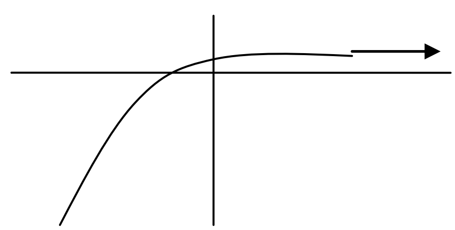
1.c. **$f(x) = (x + 2) / (x - 2)$**
Rational: y-int = -1; x-int = -2 .
Dom = {$x \neq 2$} ; Range = {$y \neq 1$}.
Ends : $x \to \pm\infty$; $y \to 1$. HorAsymp: y=1 .
VertAsymp: (divide by 0) x=2 .



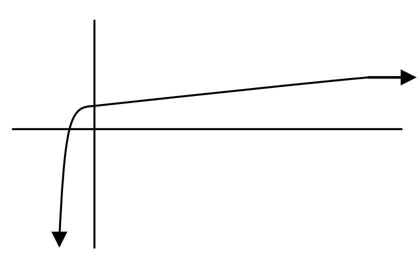
1.d. **$f(x) = (x^2 - 1) / (x^2 + 2)$**
Rational: y-int = $^{-1}/_{2}$; x-int = ±1; Dom={all x}.
Ends: $x \to \pm\infty$; $y \to 1$. HorAsymp: y=1 .
VertAsymp: none. Range={$-0.5 \leq y < 1$}.



1.e. **$f(x) = 2 - 2^{-(x+1)}$**
Exponential: y-int = 3/2 ; x-int = -2. Dom={all x}.
Ends : $x \to \infty$; $y \to 2$. HorAsymp: y=2 .
: $x \to -\infty$; $y \to -\infty$. Range={y < 2} .



1.f. **$f(x) = 1 + \log_2(x+1)$**
Logarithmic: Dom={x>-1}; y-int = 1; x-int = $^{-1}/_{2}$
VertAsymp: x = -1. Range={all y} .



2. Evaluate each of the following (single complex number, **a + bi**) :

2.a $(2 + i)^2(3 - 2i)$

(Do Foil twice):

$= (4 + 4i + i^2)(3 - 2i)$

$= (3 + 4i)(3 - 2i)$

$= (9 - 6i + 12i - 8i^2)$

$= 17 + 6i$

2.b. $(2 - 3i) \div (1 + 3i)$

$= \dfrac{(2 - 3i)(1 - 3i)}{(1 + 3i)(1 - 3i)}$

$= \dfrac{2 - 6i - 3i + 9i^2}{1 + 9}$

$= \dfrac{-7 - 9i}{10}$

3. Divide polynomials:

3.a.

```
           x² + x   + 1
x+2    | x³ + 3x² + 3x – 7
         x³ + 2x²
              x² + 3x
              x² + 2x
                   x – 7
                   x + 2
   remainder =    -9
```

3.b.

```
                    x – 2
x² + 2x – 1   | x³ + 0x² + 3x – 7
                x³ + 2x² – x
                    -2x² + 4x – 7
                    -2x² – 4x + 2
   remainder =            8x – 9
```

4. Find a function which satisfies each description:

4.a. Quadratic function with vertex = **(-2,-5)** and y-int = **4:**

Know: $y = a(x - -2)^2 + -5$

And when x=0, $4 = a(-2)^2 + -5$; so $a = {}^9/_4$.

$y = Q(x) = ({}^9/_4)(x+2)^2 - 5 = ({}^9/_4)x^2 + 9x + 4$

4.b. Polynomial function with x-int = **-2, 1, & 3,** y-int = **-6:**

By x-int's, has factors $(x - -2)(x - 1)(x - 3) = (x+2)(x-1)(x-3)$.

But this has y-int = $2(-1)(-3) = 6$, so multiply by **-1 :**

$y = P(x) = -(x+2)(x-1)(x-3) = -x^3 + 2x^2 + 5x - 6$.

5. Evaluate each of the following as a single real number:

a. **log(0.1)** = -1

b. **$\ln(e^{\pi}) = \pi$**

c. **$\log_3(10)$** = log(10)/log(3) ≈ 2.096

d. **$\log_{10}(100) = 2$**

e. **$\log_3(\sqrt{3}) = 1/2$**

f. **$3\log_2(8) – \log_4(8) =$**
$3 \cdot 3 – \log_2(8)/\log_2(4) =$
$9 – {}^{3}/_{2} = {}^{15}/_{2}$

g. **$\log_2(80) – \log_2(10) = \log_2({}^{80}/_{10}) = 3$**

h. **$\log(10{,}000{,}000{,}000{,}000) = \log(10^{13}) = 13$**

More ….

i. **$\log(0.00000000001) = \log(10^{-11}) = -11$**

j. **$\log(\text{googol}) = \log(10^{100}) = 100$**

k. **$\log(\text{googolplex}) = \log(10^{\text{googol}}) = \text{googol}$**

6. Solve each of the following for x:

6.a. **$2^{(2x+3)} = 32$**
$2x + 3 = 5$ [$\log_2$ of BS]
$2x = 2$
$x = 1$

6.b. **$2\log_2(x^2+2) = 6$**
$\log_2(x^2+2) = 3$
$x^2+2 = 2^3 = 8$ [2^BS]
$x^2 = 6$ or $x = \pm\sqrt{6}$

7) If a certain isotope has a half-life of **21** years, what percent of it decays each year? How much is left after **10** years?

The half-life amount after t years must be: **$A(t) = A(0)2^{-t/21}$** ;
Fraction left after 1 year = **$A(1)/A(0) = 2^{-1/21}$** ≈ 0.9675
Percent Left ≈ 96.75% ; Percent decays = **3.25%** .
Left after **10 = $A(10) = A(0)2^{-10/21}$ ≈ 0.7189A(0); So 71.9%.**

8) You buy a bank CD for **\$2000** which pays **6%** compounded monthly for **5** years. What is it worth at the end of the **5** years? What would it be worth if compounded continuously?

Annual: $B(5) = 2000(1+0.06)^5 = 2676.45$;
Monthly: $B(5) = 2000(1+0.06/12)^{5*12} = 2000(1.005)^{60} = 2697.70$
Continuously: $B(5) = 2000e^{(0.06)5} = 2699.72$

9) What is the doubling time and the annual rate of growth for the function:

$$\mathbf{A(t) = A_0\, e^{0.28t}}$$

The doubling time population must be: $\mathbf{A_0\, 2^{t/d}} = \mathbf{A_0\, e^{0.28t}}$
The annual growth population must be: $\mathbf{A_0(1 + r)^t} = \mathbf{A_0\, e^{0.28t}}$
Solving first: $\mathbf{A_0\, e^{0.28t} = A_0\, 2^{t/d}}$; cancel $\mathbf{A_0}$ and set **t=1**:

$$\mathbf{e^{0.28} = 2^{1/d}}$$

[ln() BS] 0.28 = (1/d)ln(2) , so, **d = ln(2)/0.28 = 2.476**
Solving second: $\mathbf{A_0\, e^{0.28t} = A_0(1+r)^t}$; cancel $\mathbf{A_0}$ and set **t=1**:

$$\mathbf{e^{0.28} = 1+r \quad , so, \quad r = 0.323}$$

10. For a certain model car, stopping distance, **d in feet**, is a function of velocity, **v in mph**, given by: $\mathbf{d(v) = 2.1v + 0.054v^2}$. For what velocity is the stopping distance **200 ft**?

So, at **30 mph**, **v(30) = 111.6**
So, at **60 mph**, **v(60) = 320.4**

If: $\mathbf{200 = 2.1v + 0.054v^2}$, this is: $\mathbf{0 = 0.054v^2 + 2.1v - 200}$ **,**
then

$$\mathbf{v = \frac{-2.1 \pm \sqrt{4.41 - 4(0.054)(-200)}}{2(0.054)} = \frac{-2.1+3.9}{0.108} = 44\,{}^4/_9 \text{ mph}}$$

11. Richter Scale for earthquakes is: $\mathbf{R = \log(I/I_s)}$ **,**
where **I** measures the force of an earthquake by the amplitude of a seismograph needle a standard distance from the quake, with $\mathbf{I_s}$ the force of a standard small quake. Basically, it takes the $\mathbf{\log_{10}}$ of the force.

So to compare forces of a **R=7.6** to **R=6.2**, take ratios of $\mathbf{10^R}$:
$\mathbf{10^{7.6}/10^{6.2} = 10^{1.4}}$ **= 25 times more forceful.**
(The destructive power is actually base **31** and not **10**: $\mathbf{31^{1.4}}$ **= 122 times.)**

11′. Decibel Scale for loudness is: $\mathbf{D = 10 \bullet \log(I/I_0)}$ **,**
where **I** measures the energy of the sound-wave front Watts/m^2, compared to a barely audible $\mathbf{I_0 = 10^{-12} W/m^2}$**.**

11″. pH scale for acidity is: $\mathbf{-\log([H+])}$ **,**
where **[H+]** is molar concentration of the **H+** ion (actually H_3O+).