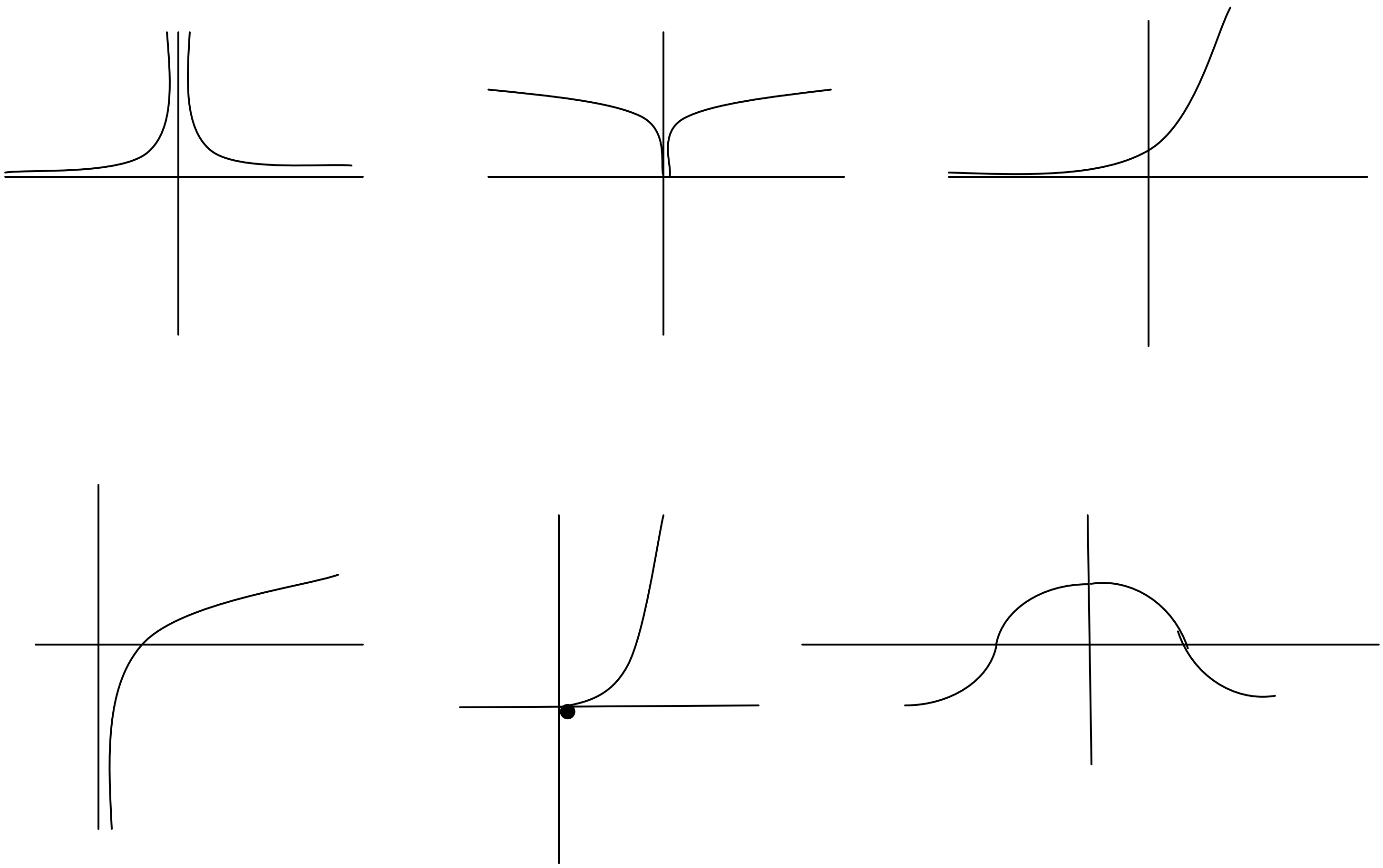
Math191 - Practice Test on Chapters 1 and 2

1. For each of the following, write a functional formula for an elementary function which would have such as its graph. Classify each as polynomial, rational, algebraic or transcendental. Classify each as odd, even or neither.



2. Write each as the composition of three elementary functions:

a) $f(x) = \sin^2(\sqrt{x}\,)$ **b)** $f(x) = \cos(\ln(x^2))$

c) $f(x) = \sin^2(x^2)$ **d)** $f(x) = \dfrac{1}{\ln(x^2)}$:

3. Graph the following and specify the domain and range: $y = \dfrac{2x^2}{x^2 + 1}$
Label the X&Y intercepts and any asymptotes

4. Consider the functions $\mathbf{f(x) = \sqrt{(x + 1)} = (x + 1)^{1/2}}$ and $\mathbf{g(x) = x^2 - 1}$.
Compute $g \circ f(x)$ and $f \circ g(x)$. Are they inverses?

5. Calculate the following limits (or state DNE if there is none):

a) $\lim_{x\to 3^+} \frac{x^2 - 9}{x^2 - 6x + 9}$

b) $\lim_{x\to 3} \frac{x^2 - 9}{x^2 + 6x + 9}$

c) $\lim_{x\to 2} \frac{x^2 + 5x - 14}{x^2 - 4}$

d) $\lim_{x\to 0} x\cot(x)$

e) $\lim_{x\to 0} \frac{1}{x} - \frac{1}{x - x^2}$

f) $\lim_{h\to 0} \frac{(1+h)^3 - 1}{h}$

g) $\lim_{x\to 0} e^{1/x}$

h) $\lim_{h\to 0} \frac{\sqrt[3]{(1+h)} - 1}{h}$

i) $\lim_{x\to +\infty} e^{1/x}$

j) $\lim_{x\to \infty} \frac{x + \sin(x)}{x}$

6. Where is each of the following NOT continuous? (May be nowhere)

a) $f(x) = \begin{cases} 0 \text{ for } x \text{ irrational} \\ 1 \text{ for } x \text{ rational} \end{cases}$

b) $f(x) = \sin(1/x)$

c) $f(x) = \sqrt{(x^2 + 1)}$

d) $f(x) = \frac{x - 1}{x^2 + x}$

7. Derive the derivative function of each by evaluating the limit of the difference quotient and then find the equation of the tangent line at **x = 0**.

a) $y = f(x) = 1/(x+1) = (x+1)^{-1}$

b) $y = f(x) = \sqrt{(x+1)} = (x+1)^{1/2}$

c) $y = f(x) = (x+1)^2$

d) $y = f(x) = x^2+3x+1$

Math191 - Practice Test Answers on Chapters 1 and 2

1. For each of the following, write a functional formula for an elementary function which would have such as its graph. Classify each as one of polynomial, rational, algebraic or transcendental. Classify each as odd, even or neither.

a) $f(x) = x^{-2}$; rational, even

b) $f(x) = x^{2/3}$; algebraic, even

c) $f(x) = e^{x}$; transcendental, none

d) $f(x) = \ln(x)$; transcendental, none

e) $f(x) = x^{3/2}$; algebraic, none

f) $f(x) = \cos(x)$; transcendental, even

2. Write each as the composition of three elementary functions:

a) $f(x) = \sin^2(\sqrt{x}\,)$: $(\ ) \to (\ )^{1/2} \to \sin(\ ) \to (\ )^2$

b) $f(x) = \cos(\ln(x^2))$: $(\ ) \to (\ )^2 \to \ln(\ ) \to \cos(\ )$

c) $f(x) = \sin^2(x^2)$: $(\ ) \to (\ )^2 \to \sin(\ ) \to (\ )^2$

d) $f(x) = \dfrac{1}{\ln(x^2)}$: $(\ ) \to (\ )^2 \to \ln(\ ) \to (\ )^{-1}$

Alternatively, **let:** $g(x) = x^2$, $h(x) = \ln(x)$, $k(x) = {}^{1}/_{x} = x^{-1}$;

then $f(x) = k \circ h \circ g\,(x)$.

OR:

$$x \xrightarrow{(\,)^2} x^2 \xrightarrow{\ln(\ )} \ln(x^2) \xrightarrow{(\ )^{-1}} \frac{1}{\ln(x^2)} \quad .$$

3.a. Graph the following and specify the domain and range: $\mathbf{y = \dfrac{2x^2}{x^2 + 1}}$
Label the X&Y intercepts and for any asymptotes

X&Y-intercept = (0,0); Domain={all x}; Range={$0 \le y < 2$}
Vertical asymptote (never divide by 0): **none**
Horizontal asymptote (limit $\lim_{x\to\pm\infty} y = 2$): **y=2**

3.b. Graph the following and specify the domain and range: $\mathbf{y = \dfrac{2x^2}{x^2 - 1}}$
Label the X&Y intercepts and for any asymptotes

X&Y-intercept = (0,0); Domain={$x \neq \pm 1$}; Range={all y}
Vertical asymptote (divide by 0 at x ±1): **x=±1**
Horizontal asymptote: **y=2**

4. Consider the functions $\mathbf{f(x) = \sqrt{x + 1} = (x + 1)^{1/2}}$ and $\mathbf{g(x) = x^2 - 1}$**.**

$$f\circ g(x) = \sqrt{(x^2 - 1) + 1} = \sqrt{x^2} = |x|$$

$$g\circ f(x) = (\sqrt{x + 1})^2 - 1 = x + 1 - 1 = x$$

So, $f\circ g(x) \neq g\circ f(x)$ and they are **not** inverses. Notice: **g(x)** is not **1-1**.

5. Calculate the following limits.

a) $$\lim_{x\to 3^+} \frac{x^2 - 9}{x^2 - 6x + 9} = \frac{0}{0} = \lim_{x\to 3^+} \frac{\cancel{(x - 3)}(x + 3)}{\cancel{(x - 3)}(x - 3)} = \frac{6}{0^+} = +\infty \text{ (DNE)}$$

b) $$\lim_{x\to 3} \frac{x^2 - 9}{x^2 + 6x + 9} = \frac{0}{36} = 0$$

c) $$\lim_{x\to 2} \frac{x^2 + 5x - 14}{x^2 - 4} = \frac{0}{0} = \lim_{x\to 2} \frac{\cancel{(x - 2)}(x + 7)}{\cancel{(x - 2)}(x + 2)} = \frac{9}{4}$$

d) $\lim_{x\to 0} x\cot(x) = 1 = \lim \frac{x\cos(x)}{\sin(x)} = \lim \frac{x}{\sin(x)} \cdot \lim \cos(x) = 1\cdot 1$

(Note: Can't yet do properly, $\lim_{x\to 0} \frac{x}{\sin x}$; so use calculator.)

e) $\lim_{x\to 0} \frac{1}{x} - \frac{1}{x - x^2} = \lim \frac{-x^2}{x^2(1 - x)} = -1$ (fish:LCD)

f) $\lim_{h\to 0} \frac{(1+h)^3 - 1}{h} = \lim \frac{1 + 3h + 3h^2 + h^3 - 1}{h} =$

$\lim \frac{h(3+3h+h^2)}{h} = 3$

g) $\lim_{x\to 0} e^{1/x} = e^{1/\pm 0} = e^{\pm\infty} = 0 \text{ or } +\infty = \text{DNE}$

h) $\lim_{h\to 0} \frac{\sqrt[3]{(1+h)} - 1}{h} = \lim \frac{[(1+h)^{1/3} - 1]\bullet[(1+h)^{2/3}+(1+h)^{1/3}+1]}{h \bullet [(1+h)^{2/3}+(1+h)^{1/3}+1]}$

$\lim_{h\to 0} \frac{(1 + h) - 1}{h[(1+h)^{2/3}+(1+h)^{1/3}+1]} = \frac{1}{[1+1+1]} = \frac{1}{3}$

i) $\lim_{x\to\infty} e^{1/x} = e^{1/\pm\infty} = e^0 = 1$

j) $\lim_{x\to\infty} \frac{x + \sin(x)}{x} = \lim \frac{x}{x} + \lim \frac{\sin(x)}{x} = 1 + 0 = 1$

(squeeze theorem: $-1 \leq \sin(x) \leq 1$ and $^{\pm 1}/_{x} \to 0$)

6. Where is each of the following NOT continuous? (May be nowhere)

a) $f(x) = \begin{cases} 0 \text{ for x irrational} \\ 1 \text{ for x rational} \end{cases}$ **not continuous everywhere**

b) $f(x) = \sin(1/x)$ **not continuous at x = 0**

c) $f(x) = \sqrt{x^2 + 1}$ **not continuous nowhere**

d) $f(x) = \dfrac{x - 1}{x^2 + x}$ **factor bottom as x(x+1) ; not continuous at x = 0, -1**

7. Derive the derivative of each by explicitly evaluating the limit of the difference quotient. Find the equation of the tangent line at x = 0.

DQ: $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$ **TL:** $y = f(0) + f'(0)(x - 0)$

(a) $y = 1/(x+1) = (x+1)^{-1}$

$f'(x) = \lim_{h\to 0} \dfrac{(x+h+1)^{-1} - (x+1)^{-1}}{h}$ (common denominator)

$= \lim_{h\to 0} \dfrac{(x+1) - (x+h+1)}{h(x+1)(x+h+1)} = \lim_{h\to 0} \dfrac{-1}{(x+1)(x+h+1)}$

$= \dfrac{-1}{(x+1)^2} = -(x+1)^{-2}$

TL: $y = 1 + -1(x - 0) = 1 - x$

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(b) $y = \sqrt{(x+1)} = (x+1)^{1/2}$

$f'(x) = \lim_{h\to 0} \dfrac{(x+h+1)^{1/2} - (x+1)^{1/2}}{h} \bullet \dfrac{[(x+h+1)^{1/2} + (x+1)^{1/2}]}{[(x+h+1)^{1/2} + (x+1)^{1/2}]}$ **radical conjugate**

$= \lim_{h\to 0} \dfrac{(x+h+1) - (x+1)}{h[(x+h+1)^{1/2} + (x+1)^{1/2}]}$

$$= \lim_{h\to 0} \frac{1}{[(x+h+1)^{1/2} + (x+1)^{1/2}]} = \frac{1}{2(x+1)^{1/2}} = \frac{1}{2\sqrt{(x+1)}} =$$

$= (^1/_2)(x+1)^{-1/2}$ TL: $y = 1 + (^1/_2)(x - 0) = \frac{x+2}{2}$

==

(c) $y = (x+1)^2$

$$f'(x) = \lim_{h\to 0} \frac{(x+h+1)^2 - (x+1)^2}{h} = \frac{(x^2 + h^2 + 1^2 + 2xh + 2x + 2h) - (x^2+2x+1)}{h}$$

$$= \lim \frac{2xh + h^2 + 2h}{h} =$$

$= 2x + 2$ TL: $y = 1 + 2(x - 0) = 2x + 1$

===

(d) $y = x^2 + 3x+1$

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^2 + 3(x+h)+1 - (x^2 + 3x+1)}{h}$$

$$= \lim \frac{2xh + h^2 + 3h}{h} =$$

$= 2x + 3$ TL: $y = 1 + 3(x - 0) = 3x + 1$